A Simplified Method for Parameter Estimating of Double Exponential Pulse

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***Abstract*—High-altitude electromagnetic pulse (HEMP), ultrawide-band pulse (UWB) and lightning electromagnetic pulse (LEMP) are often mathematically described by a double exponential function. In practice, the physical parameters of the pulse and the mathematical parameters of the function must often be transformed into each other. In this paper, a strict implicit translation equation group of parameters is deduced. Meanwhile, a simplified translation system of equations for parameter estimation is proposed based on statistical method. Two groups of correction terms are also given to improve the estimation precision. The estimation results demonstrate that the overall estimation error is less than 2.9%.**

Keywords*—*double exponential function; parameter estimation; high-altitude electromagnetic pulse (HEMP); ultrawide-band pulse (UWB); lightning electromagnetic pulse (LEMP);

I. INTRODUCTION

Double exponential function is widely used to mathematically describe high-altitude electromagnetic pulse (HEMP), ultrawide-band pulse (UWB) and lightning electromagnetic pulse (LEMP). This type of function contains four mathematical parameters: α, β, k and E_0. Associated with the parameters of the function, there are three physical parameters that describe the pulse shape: the rise time (t_r), the full width at half maximum amplitude (t_{FWHM}) and the maximum electrical field strength (E_{max}). The two groups of parameters often need to be transformed into each other. In particular, researchers pay close attention to the relationships among α, β and t_r, t_{FWHM}. A method has been proposed to estimate the physical parameters using least squares and Nelder-Mead algorithms, and inverse transform equations can be given by the same algorithms [2]. This approach gives an integrated system of transform equations, but the equations are not simple or direct. Moreover, the overall estimation error of this method is greater than 5%. To improve the precision, a linear equation group with four assistant variables has been proposed based on statistical methods [3]. Different variables are given for different intervals of β/α. Based on the same method, another equation group is given with several linear functions for larger values of β/α and two offset exponential functions for smaller values of β/α [4], which gives the number of the section. The use of statistical methods improves the estimation precision, but the function is somewhat complicated.

In this paper, a strict implicit translation equation group of the parameters is deduced. With the goals of simplicity and precision, a simplified translation system of equations is proposed with correction terms for parameter estimation based on the statistical method described in [3] and [4]. Finally, the estimation errors are analysed.

II. SHAPE PROPERTIES OF DOUBLE EXPONENTIAL PULSE

The double exponential pulse can be described as follows:

$$E(t) = E_0 k(\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\beta t}) \cdot u(t) \tag{1}$$

where E_0 is the amplitude of the pulse, α and β are parameters characterizing the rise and fall time, and $u(t)$ is a step unit function. A fourth parameter k can be calculated by

$$k = \mathrm{e}^{-\alpha\frac{\ln\alpha - \ln\beta}{\alpha-\beta}} - \mathrm{e}^{-\beta\frac{\ln\alpha - \ln\beta}{\alpha-\beta}} \tag{2}$$

The definitions of the rise time (t_r) and pulse width (t_{FWHM}) are shown in Fig. 1. The rise time corresponds to the time period from 10% to 90% of the maximum value. The pulse width is the time interval between the 50% value at the rising edge and the 50% value at the falling edge.

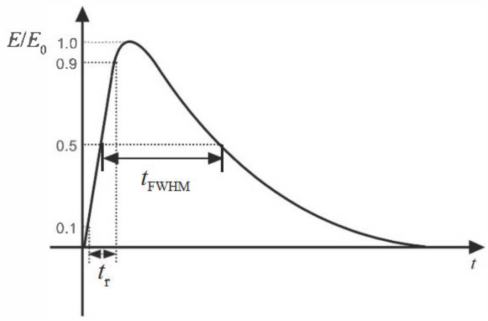


Fig. 1. Pulse shape and the definition of t_r and t_{FWHM}

III. STRICT EQUATIONS FOR TWO GROUPS OF PARAMETERS

Based on the definition of t_r, we can obtain (3):

$$t_{90\%} = t_{10\%} + t_r \tag{3}$$

where $t_{10\%}$ and $t_{90\%}$ denote the time at which the magnitude equals 10% and 90% of E_0. The values of $t_{10\%}$ and $t_{90\%}$ satisfy (4) and (5):

$$k(\mathrm{e}^{-\alpha t_{10\%}} - \mathrm{e}^{-\beta t_{10\%}}) = 0.1 \tag{4}$$

$$k(\mathrm{e}^{-\alpha t_{90\%}} - \mathrm{e}^{-\beta t_{90\%}}) = 0.9 \tag{5}$$

From (3) and (5),

$$k(\mathrm{e}^{-\alpha(t_{10\%}+t_r)} - \mathrm{e}^{-\beta(t_{10\%}+t_r)}) = 0.9 \tag{6}$$

$$(6)-(4)\times \mathrm{e}^{-\beta t_r}$$

$$k\mathrm{e}^{-\alpha t_{10\%}}(\mathrm{e}^{-\alpha t_r} - \mathrm{e}^{-\beta t_r}) = 0.9 - 0.1\mathrm{e}^{-\beta t_r} \tag{7}$$

$$(6)-(4)\times \mathrm{e}^{-\alpha t_r}$$

$$k\mathrm{e}^{-\beta t_{10\%}}(\mathrm{e}^{-\alpha t_r} - \mathrm{e}^{-\beta t_r}) = 0.9 - 0.1\mathrm{e}^{-\alpha t_r} \tag{8}$$

Thus, the parameter t_r can be eliminated by (7) and (8). The relationship among α, β, t_r can be written as (9):

$$\left[\frac{k(\mathrm{e}^{-\alpha t_r} - \mathrm{e}^{-\beta t_r})}{0.9-0.1\mathrm{e}^{-\beta t_r}}\right]^{\beta} = \left[\frac{k(\mathrm{e}^{-\alpha t_r} - \mathrm{e}^{-\beta t_r})}{0.9-0.1\mathrm{e}^{-\alpha t_r}}\right]^{\alpha} \tag{9}$$

In the same way, the relationship among α, β, t_{FWHM} can be deduced as (10):

$$\left[\frac{k(\mathrm{e}^{-\alpha t_{FWHM}} - \mathrm{e}^{-\beta t_{FWHM}})}{0.5-0.5\mathrm{e}^{-\beta t_{FWHM}}}\right]^{\beta} = \left[\frac{k(\mathrm{e}^{-\alpha t_{FWHM}} - \mathrm{e}^{-\beta t_{FWHM}})}{0.5-0.5\mathrm{e}^{-\alpha t_{FWHM}}}\right]^{\alpha} \tag{10}$$

The translation equations for the mathematical parameters α, β and the physical parameters t_r, t_{FWHM} are described by (9) and (10). Obviously, these equations are transcendental, indicating that strict explicit translation equations do not exist.

IV. Estimation Between Two Groups of Parameters

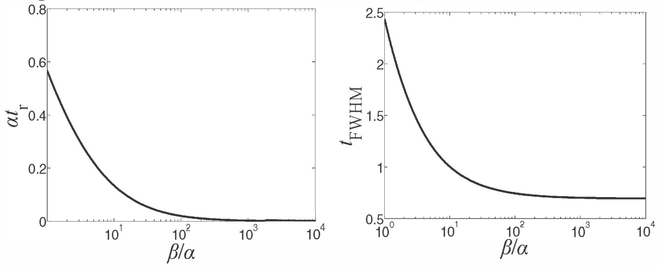
It has been confirmed that αt_r and αt_{FWHM} are constant for a given ratio of β/α [3]. Based on the definitions of t_r and t_{FWHM}, a highly precise value of αt_r and αt_{FWHM} can be calculated via iterative algorithms. Solutions of αt_r and αt_{FWHM} with an accuracy of 15 effective digits are plotted in Fig. 2, where β/α ranges from 1.001 to 10000. Interestingly, the values between $\alpha^2 t^2_{FWHM}$ and α/β have a nearly linear relationship, and the products of $\alpha\beta t_r t_{FWHM}$ are approximately constant, as shown in Fig. 3.

Based on linear fitting, the data shown in Fig. 3 can be fit as (11):

$$\alpha^2 t^2_{FWHM} = 5.4\frac{\alpha}{\beta} + 0.485 + \Delta_1$$

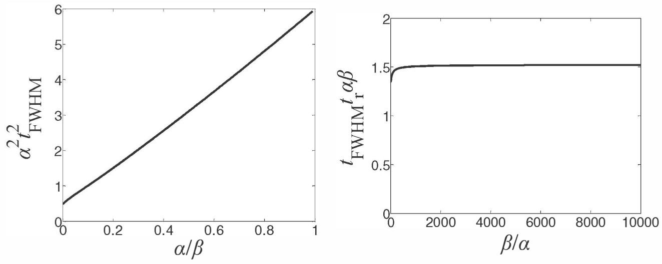
$$t_{FWHM}\cdot t_r \cdot \alpha \cdot \beta = 1.505 + \Delta_2 \tag{11}$$

where Δ_1 and Δ_2 are correction terms used to improve the estimation precision. Assuming that Δ_1 and Δ_2 are constant, the parameters can be calculated from (11).



(a) Parameters αt_r versus β/α (b) Parameters αt_{FWHM} versus β/α

Fig. 2. Parameters αt_r and αt_{FWHM} for β/α ranging from 1.001 to 10000.



(a) Variations in parameter $\alpha^2 t^2_{FWHM}$ (b) Variations in parameter $\alpha^2 t^2_{FWHM}$

Fig. 3. Variations in parameters $\alpha^2 t^2_{FWHM}$ and $\alpha\beta t_r t_{FWHM}$.

The parameters t_r and t_{FWHM} can be calculated as (12) when α and β are known.

$$\begin{aligned} t_{FWHM} &= \frac{1}{\alpha}\sqrt{5.4\frac{\alpha}{\beta}+0.485+\Delta_1} \\ t_r &= \frac{1}{t_{FWHM}}\frac{1.505+\Delta_2}{\alpha\beta} \end{aligned} \tag{12}$$

The parameters α and β can also be calculated, as shown in (13), when t_r and t_{FWHM} are known.

$$\begin{aligned} \alpha &= \frac{1}{t_{FWHM}}\sqrt{\frac{(0.485+\Delta_1)(1.505+\Delta_2)\frac{t_{FWHM}}{t_r}}{(1.505+\Delta_2)\frac{t_{FWHM}}{t_r}-5.4}} \\ \beta &= \frac{1}{\alpha}\frac{1.505+\Delta_2}{t_{FWHM}t_r} \end{aligned} \tag{13}$$

To obtain certain parameter values from equation (11), Δ_1 and Δ_2 must be constant, indicating that Δ_1 and Δ_2 should be described by functions that are independent of the parameters to be calculated. Thus, we can divide our approach into two conditions:

- Condition 1. α and β are known.
- Condition 2. t_r and t_{FWHM} are known.

The sum of the two exponential functions is used to fit the correction terms for varying values of α/β under condition1 and for varying values of t_{FWHM}/t_r under condition 2. The fitting results for Δ_1 and Δ_2 are given by (14) and (15), respectively.

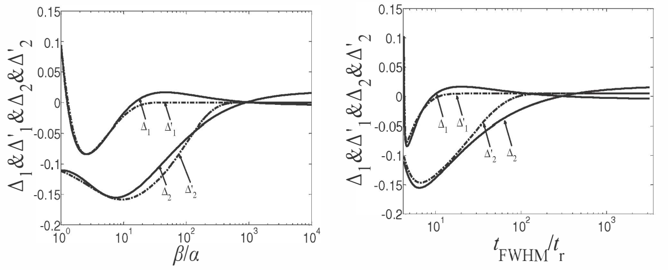
$$\Delta_1 = \begin{cases} 1.8e^{-2.1\frac{\beta}{\alpha}} - 0.15e^{-0.19\frac{\beta}{\alpha}} \\ -0.6461e^{-0.4394\frac{t_{FWHM}}{t_r}} + 3.2993\times10^{+23}e^{-13\frac{t_{FWHM}}{t_r}} + 0.005 \end{cases} \quad (14)$$

$$\Delta_2 = \begin{cases} 0.09e^{-0.3\frac{\beta}{\alpha}} - 0.18e^{-0.01\frac{\beta}{\alpha}} \\ -0.2066e^{-0.03982\frac{t_{FWHM}}{t_r}} + 4.015e^{-0.9509\frac{t_{FWHM}}{t_r}} + 0.005 \end{cases} \quad (15)$$

Fig. 4 presents the fitting results for Δ_1 and Δ_2 versus α/β and t_{FWHM}/t_r, respectively, where Δ'_1 and Δ'_2 are accuracy values of the correction terms calculated from (16) and (17).

$$\Delta'_1 = \alpha^2 t_{FWHM}^2 - 5.4\frac{\alpha}{\beta} - 0.485 \quad (16)$$

$$\Delta'_2 = t_{FWHM} \cdot t_r \cdot \alpha \cdot \beta - 1.505 \quad (17)$$



(a) The correction terms versus α/β (b) The correction terms versus t_{FWHM}/t_r

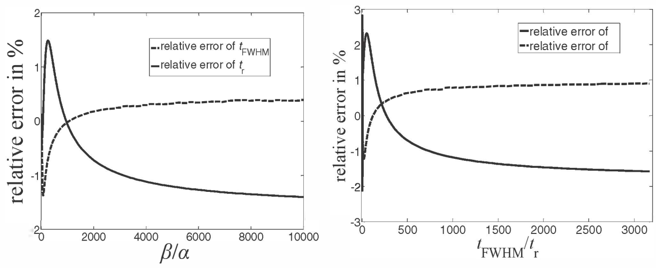
Fig. 4. The fitting results of Δ_1 and Δ_2 compared with the accuracy values of the correction terms versus α/β and t_{FWHM}/t_r.

V. Analysis of the Estimation Errors

Obviously, for a given value of β/α or t_{FWHM}/t_r, the relative errors for estimation are the same. The estimation error curves are shown in Fig. 5, where β/α ranges from 1.001 to 10000. Corresponding to β/α, t_{FWHM}/t_r ranges from 4.291 to 3169. Based on these results, the estimation errors of t_{FWHM} and t_r are 1.5% and 1.4%, respectively, and the estimation errors of β and α are less than 1.3% and 2.9%.

In addition, we also examine the relative errors of the reproduction terms t'_{FWHM} and t'_r, corresponding to the α and β values estimated by (12), compared with the known t_{FWHM}, t_r. As shown in Fig. 6, the estimation errors of t'_r and t'_{FWHM} are less than 2.1% and 1%.

This paper confirms that the presented method produces limited parameter estimation errors. However, based on the tendency of these error curves, we believe that the relationships between α, β and t_r, t_{FWHM} could be used for larger ratios of β/α and t_{FWHM}/t_r.



(a) Relative errors of t_{FWHM} and t_r (b) Relative estimation errors of α and β

Fig. 5. Relative estimation errors of t_{FWHM} and t_r versus β/α and relative estimation errors of α and β versus t_{FWHM}/t_r.

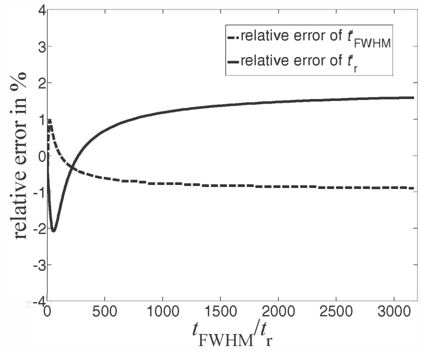


Fig. 6. Relative errors of the reproduction terms t'_{FWHM} and t'_r, corresponding to the estimated α and β, compared with the known t_{FWHM}, t_r.

VI. Conclusion

This paper gives a strict implicit translation equation group of parameters for double exponential function. Using statistical method, a simplified approximate translation system of equations with correction terms is proposed, which can translate the mathematical parameters α and β and the physical parameters t_r and t_{FWHM} into each other. With this method, the estimation errors of t_r and t_{FWHM} are less than 1.5% and 1.4%, respectively, and the estimation errors of α and β are less than 1.3% and 2.9%, respectively. In addition, the estimation error of the reproduction terms t'_r and t'_{FWHM}, corresponding to the estimated values of α and β, are less than 2.1% and 1%, respectively.

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