

## V. CONCLUSION

The described method of calibration proved to be simple and accurate. It is particularly suitable for automatic operation. A calibrated standard can be easily measured versus a number of reference standards, and this makes results very accurate and credible. Additionally all standards of the reference group are mutually checked in each calibration process.

## ACKNOWLEDGMENT

The author would like to thank George Free and Charles Levy of the National Institute of Standards and Technology for their invaluable help throughout this project.

## REFERENCES

- [1] T. L. Zapf, "Calibration of inductance standards in the Maxwell-Wien bridge circuit," *J. Res. NBS*, vol. 65C, pp. 183-188, Sept. 1961.
- [2] A. Muciek, "Precise inductance measurements at low frequencies," (in Polish) Wrocław, Technical University, Poland, 1985.
- [3] D. L. Hillhouse, "Resonant capacitance-inductance transfer using the ratio transformer," *IEEE Trans. Commun. Electron.*, vol. 82, pp. 158-164, Jan. 1963.
- [4] A. F. Dunn and S. H. Tsao, "Ratio comparisons of impedance standards," *IEEE Trans. Instrum. Meas.*, vol. IM-15, pp. 276-283, 1965.
- [5] A. Muciek, "A combined transformer bridge for precise comparison of inductance with capacitance," *IEEE Trans. Instrum. Meas.*, vol. IM-32, Sept. 1983.
- [6] D. L. Hillhouse and H. W. Kline, "A ratio transformer bridge for standardization of inductors and capacitors," *IRE Trans. Instrum.*, vol. I-9, pp. 251-257, Sept. 1960.
- [7] E. Arri and G. Noce, "A high accuracy self-calibrating bridge with coupled inductive ratio arms used for standard inductors comparison," *IEEE Trans. Instrum. Meas.*, vol. IM-23, pp. 72-79, Mar. 1974.
- [8] G. M. Free, "A high accuracy, easy to use, 100 Hz to 100 kHz inductance bridge," *CPEM '84 Conf. Dig.*, Delft, The Netherlands, Aug. 20-24, 1984.
- [9] A. Muciek, "A least squares method for analysis of pair comparison measurements," *IMEKO-Symp. on Measurement and Estimation*, Bressanone, Italy, May 7-11, 1984.
- [10] K. Neumann, "Verbesserte Präzisionsmeßeinrichtung zur Darstellung der Einheit der Induktivität im ASMW," *Metrologische Abhandlungen 4*, pp. 129-135, 1984.
- [11] —, "Das Temperaturverhalten von Induktivitätsnormalen verschiedener Nennwerte," *Metrologische Abhandlungen 7*, pp. 55-63, 1987.
- [12] R. Hanke and K. Dröge, "Calculated frequency characteristic of GR1482 inductance standards between 100 Hz and 100 kHz," *IEEE Trans. Instrum. Meas.*, vol. 40, pp. 72-79, 1991.
- [13] H. Eckardt and K. Neumann, "Present status of the international comparison of 10 mH inductance standards," CCE doc. CCE/92-41b.
- [14] A. Fiebiger, R. Hanke and K. Dröge, "International intercomparison of inductance standards 1982-1985," *PTB-Bericht Ptb-E-32*, Braunschweig, July 1988.

## On the Design and Generation of the Double Exponential Function

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**Abstract**—For the double exponential function  $f(t) = K(e^{-at} - e^{-bt})$ , which is used for impulse testing of electrical components and systems, we derive an approximate relation between the ratio  $y_m = T_m/T_{\max}$ , where  $T_{\max}$  and  $T_m$  are, respectively, the times to reach the peak value  $F_{\max}$  and the value  $F_{\max}/m$  on the tail of the pulse, and the ratio  $x = b/a$ . This relation is useful for finding  $x$  for a prescribed  $y_m$ , where  $m$  is usually equal to 2. Our formula is much simpler than that given by Googe, Ewing and Hess [1], but gives results of comparable accuracy. We also present a number of RC two-ports, alternative to those of [1], for generating the test function  $f(t)$  from an impulse function  $\delta(t)$ , as well as from the step function  $u(t)$ .

## I. INTRODUCTION

In a recent paper [1], Googe, Ewing and Hess considered the double exponential function

$$f(t) = K(e^{-at} - e^{-bt}) \quad (1)$$

which is often used in testing electrical devices and systems for their response to lightning and switching transients and electromagnetic pulse disturbances, and derived the following relationships connecting the parameter  $x = b/a$ ,  $T_{\max}$  = time at which  $f(t)$  reaches the maximum value  $F_{\max}$  and  $T_x$  = time at which  $f(t)$  attains the value  $F_{\max}/2$  on the tail of the pulse (see Fig. 1)

$$aT_{\max} = (\ln x)/(x-1) \quad (2)$$

$$y = \frac{T_x}{T_{\max}} \approx \frac{-\ln \{0.5 [e^{-(\ln x)/(x-1)} - e^{-x(\ln x)/(x-1)}]\}}{(\ln x)/(x-1)} \quad (3)$$

While (2) is an exact relationship, (3) is an approximate one<sup>1</sup>, based on the practical situation that  $b \gg a$ , i.e.  $x \gg 1$ . The latter equation has to be solved for  $x$  for a prescribed  $y$ . An iterative procedure for the purpose has been given in [1], based on the observation that a first guess for  $x$  is  $x \cong 10y$ .

In this paper, we first derive a relation between  $y$  and  $x$ , which is simpler than (3). We next carry out a detailed study of the problem of synthesizing an RC two-port which will generate  $f(t)$  in response to an excitation of the form of  $\delta(t)$  or  $u(t)$ , where  $\delta(t)$  and  $u(t)$  denote the unit impulse and unit step function, respectively.

II. DERIVATION OF THE RELATION BETWEEN  $y$  AND  $x$ 

Maximizing (1) gives the condition

$$e^{-bT_{\max}} = (1/x)e^{-aT_{\max}} \quad (4)$$

This is the same as (2), expressed in a different form. Now from (1) and (4),

$$F_{\max} = f(T_{\max}) = Ke^{-aT_{\max}}[1 - (1/x)] \quad (5)$$

Manuscript received August 12, 1994.

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Publisher Item Identifier S 0018-9456(96)00859-5.

<sup>1</sup>It may be mentioned in passing that (9) of [1] is wrong. Equation (3) as given here is the correct form.

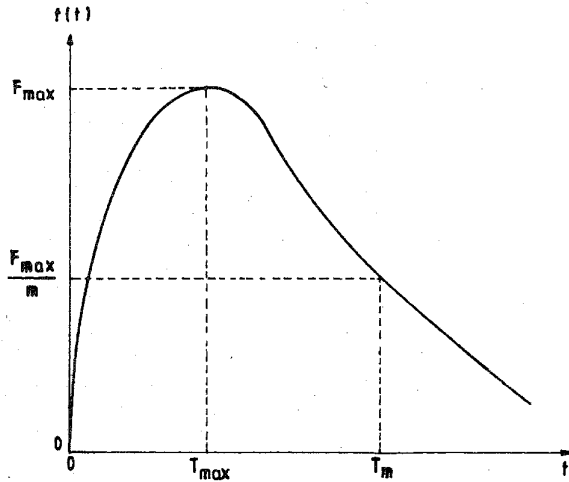


Fig. 1. Plot of the double exponential function (1). The authors of [1] considered  $m = 2$  and used  $T_x$  for  $T_2$ .

Let  $T_m$  be defined by  $f(T_m) = F_{\max}/m$  at the tail end of (1). This is a generalization of  $T_x$  of [1], for which  $m = 2$ . Thus

$$e^{-aT_m} - e^{-bT_m} \cong e^{-aT_m} = e^{-aT_{\max}} [1 - (1/x)]/m. \quad (6)$$

where we have used the fact that  $b \gg a$  so that  $e^{-aT_m} \gg e^{-bT_m}$ . From (6), we get

$$y_m = T_m/T_{\max} = 1 + \{\ln[mx/(x-1)]\}/(aT_{\max}). \quad (7)$$

In (7), if we substitute the value of  $aT_{\max}$  from (2) and use the approximation  $x \gg 1$ , then we get

$$y_m = 1 + x(\ln m)/(\ln x). \quad (8)$$

For the half-peak case, we have  $m = 2$  so that

$$y_2 = 1 + 0.693x/(\ln x). \quad (9)$$

This is a much simpler equation than (3). Computation shows that for  $x = 10, 100$  and  $1000$ ,  $y_2 = 4.01, 16.05$  and  $101.34$  respectively, the corresponding values of  $y$  (from (3)) being  $4.12, 16.12$  and  $101.38$ . The difference therefore lies only in the fractional part, and it decreases with increasing  $x$ .

### III. RC TWO-PORT SYNTHESIS FOR GENERATING (1) FROM A UNIT IMPULSE EXCITATION

As noted in [1], if a two-port is to produce (1) at the output with a unit impulse as the excitation, then its transfer function should be

$$G(s) = \mathcal{L}[f(t)]/\mathcal{L}[\delta(t)] \\ = K(b-a)/[(s+a)(s+b)]. \quad (10)$$

Identifying this as  $-y_{21}/y_{22}$ , and taking  $y_{22}$  as  $(s+a)(s+b)/(s+c)$ , where  $a < c < b$ , the authors of [1] obtained the ladder realization of Fig. 2, and claimed that the choice of  $c$  would affect the gain of the two-port and hence  $K$ . This was justified through another network, shown in Fig. 3, which is substantially different from the network of Fig. 2 in architecture as well as excitation. We shall deal with both of these networks as well as some possible alternative designs in detail.

First, look at Fig. 2 and note that the d.c. gain of the network,  $G(0)$ , is unity, so that from (10),

$$K = ab/(b-a). \quad (11)$$

This is independent of  $c$ , and so is the output

$$v_o(t) = [ab/(b-a)](e^{-at} - e^{-bt}) \quad (12)$$

contrary to the assertion made in [1].

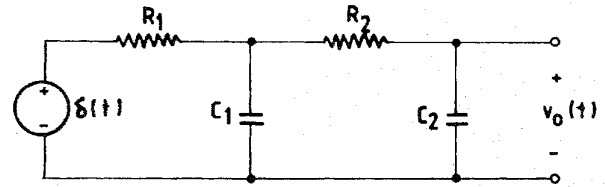


Fig. 2. Ladder network for realizing  $f(t)$ .

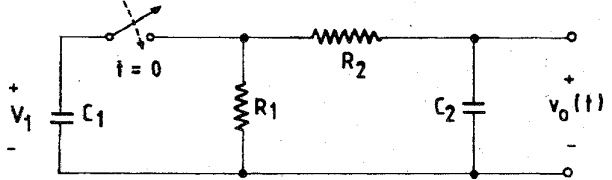


Fig. 3. Impulse generating network used in [1] to justify an optimum choice of the parameter  $c$ .

Straightforward analysis of the Fig. 2 network gives

$$G(s) = (C_1 C_2 R_1 R_2)^{-1} / \\ \{s^2 + s[(C_2 R_2)^{-1} + (C_1 R_1)^{-1} + (C_1 R_2)^{-1}] \\ + (C_1 C_2 R_1 R_2)^{-1}\} \\ = ab/[s^2 + s(a+b) + ab] \quad (13)$$

where the latter form follows from a combination of (10) and (11). Comparing the coefficients of the two forms, we get

$$(C_1 C_2 R_1 R_2)^{-1} = ab \quad (14)$$

$$(C_2 R_2)^{-1} + (C_1 R_1)^{-1} + (C_1 R_2)^{-1} = a + b. \quad (15)$$

Since there are four elements, and only two constraints, we are free to choose two elements as per convenience and calculate the other two in terms of these. Accordingly, several designs are possible, as illustrated by the following special cases.

*Case 1—Equal Capacitor Design:* Let  $C_1 = C_2 = C$ ; then (14) and (15) give

$$G_1 G_2 = abC^2 \quad (16)$$

$$G_1 + 2G_2 = (a+b)C \quad (17)$$

where  $G_i = 1/R_i$ ,  $i = 1, 2$ . Combining (16) and (17), one can obtain a quadratic equation in  $G_2$  (or in  $G_1$ ), the solution of which is

$$G_2 = [(a+b)C/4]\{1 \pm [1 - 8ab/(a+b)^2]^{1/2}\}. \quad (18)$$

Thus  $G_2$  has two possible values, provided, of course, that

$$8ab < (a+b)^2. \quad (19)$$

Equation (19) can be put in the form

$$[b - (3 + \sqrt{8})a][b - (3 - \sqrt{8})a] > 0. \quad (20)$$

Since, in practice,  $b/a \gg 1$ , the only acceptable condition for satisfying (20) becomes

$$b/a > (3 + \sqrt{8}) = 5.83. \quad (21)$$

With the notation

$$d = [1 - 8ab/(a+b)^2]^{1/2} \quad (22)$$

we can now summarize the equal capacitor design as

$$C_1 = C_2 = C$$

$$b/a > 5.83$$

$$R_1 = (a+b)(1 \pm d)/(4abC)$$

$$R_2 = (a+b)(1 \mp d)/(2abC). \quad (23)$$

Case 2—Equal Resistor Design: With  $R_1 = R_2 = R$ , (14) and (15) give

$$D_1 D_2 = abR^2 \quad (24)$$

$$D_2 + 2D_1 = (a + b)R \quad (25)$$

where  $D_i = 1/C_i$ ,  $i = 1, 2$ . The similarity of these equations with (16) and (17) allows us to write down the design equations without further analysis as

$$\begin{aligned} R_1 &= R_2 = R \\ b/a &> 5.83 \end{aligned} \quad (26)$$

$$C_1 = (a + b)(1 \mp d)/(2abR)$$

$$C_2 = (a + b)(1 \pm d)/(4abR).$$

Case 3—Design Given by Googe, Ewing and Hess [1]: In [1],  $C_2$  was normalized to 1 F and  $R_2$  came out to be  $1/(a + b - c)$ , where  $a < c < b$ . Thus, from (14) and (15), we get

$$(C_1 R_1)^{-1} = ab/(a + b - c) \quad (27)$$

$$(R_1^{-1} + a + b - c)C_1^{-1} = c. \quad (28)$$

Solving these equations, we get

$$C_1 = (a + b - c)^2/[c(a + b - c) - ab] \quad (29)$$

$$R_1 = [c(a + b - c) - ab]/[ab(a + b - c)].$$

These are the same values as given in [1].

#### IV. NETWORK SYNTHESIS FOR GENERATING $f(t)$ FROM A UNIT STEP FUNCTION

We now consider the network of Fig. 3, where  $C_1$  is first charged to the d.c. voltage  $V_1$ , and then the switch is closed at  $t = 0$ . By Thevenin's theorem, this network is equivalent to that of Fig. 4. Clearly, this is a bandpass network (in contrast to Fig. 2, which is a lowpass network) excited by a step function voltage  $V_1 u(t)$ . By elementary analysis, the transfer function for this network is obtained as

$$G_2(s) = (C_2 R_2)^{-1} s / \{s^2 + [(C_2 R_2)^{-1} + (C_1 R_2)^{-1}] + (C_1 R_1)^{-1}\} s + (C_1 C_2 R_1 R_2)^{-1}. \quad (30)$$

When excited by a unit step voltage  $u(t)$  ( $V_1$  is assumed to be unity, for convenience and without any loss of generality), the output voltage transform would be

$$\begin{aligned} V_o(s) &= (1/s)G_2(s) \\ &= (C_2 R_2)^{-1} / \{s^2 + [(C_2 R_2)^{-1} + (C_1 R_1)^{-1}] \\ &\quad + (C_1 R_2)^{-1}\} s + (C_1 C_2 R_1 R_2)^{-1}. \end{aligned} \quad (31)$$

Clearly, this is of the form (10) with

$$\begin{aligned} K(b - a) &= (C_2 R_2)^{-1} \\ (C_2 R_2)^{-1} + (C_1 R_1)^{-1} + (C_1 R_2)^{-1} &= a + b \\ (C_1 C_2 R_1 R_2)^{-1} &= ab. \end{aligned} \quad (32)$$

Hence, as claimed correctly in [1],  $v_o(t)$  will be of the form (1). If the same elements as those for the lowpass network of [1] (which is the same as Case 3 of the previous section) are used, then we get

$$K = 1/[C_2 R_2(b - a)] = (a + b - c)/(b - a). \quad (33)$$

This shows that indeed, the peak value of the generated pulse now depends on  $c$ , and  $c = 0$  gives the highest value. However, because  $a < c < b$ , and  $b \gg a$ ,  $c = 2a$  is quite an acceptable value, giving  $K = 1$ .

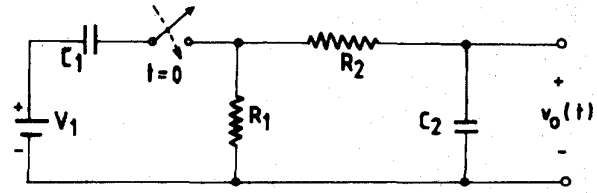


Fig. 4. Thevenin equivalent of the network of Fig. 3.

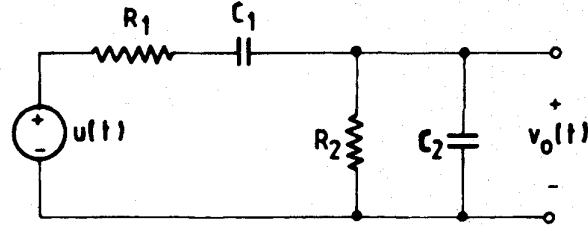


Fig. 5. A possible network realization of (36).

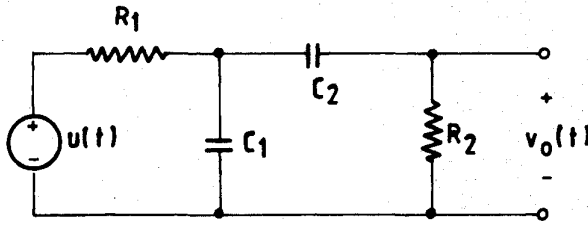


Fig. 6. Another candidate network for realizing (36).

For the element values of Case 1 of the preceding section (equal capacitor design), we have from (23) and (32),

$$K = (b + a)(1 \pm d)/[4(b - a)]. \quad (34)$$

Using the positive sign in (34) gives a higher  $K$ ; this corresponds to the negative sign in  $R_2$  given by (23). The highest possible value of  $K$  is obtained when  $b \gg a$  and  $b \gg 8$ , and this value is  $1/2$ , which is half of that obtained from the design of [1].

For the Case 2 design (with equal resistors), we get from (26) and (32),

$$K = (b + a)(1 \mp d)/[2(b - a)]. \quad (35)$$

Once again, the positive sign gives a higher value of  $K$ , which corresponds to the negative sign in  $C_2$  given by (26). The highest possible value of  $K$  here is unity (with  $b \gg a$  and  $b \gg 8$ ), in contrast to the Case 1 design. Hence, equal resistor design is to be preferred compared to equal capacitor design. Compared to both of these designs, however, the design of Googe, Ewing and Hess [1] has an edge with respect to the peak voltage obtained.

The bandpass transfer function of the form

$$G_2(s) = K(b - a)s/[(s + a)(s + b)] \quad (36)$$

can also be realized by either of the two networks shown in Figs. 5 and 6. For the former,

$$\begin{aligned} (C_2 R_1)^{-1} &= K(b - a) \\ (C_1 R_1)^{-1} + (C_2 R_2)^{-1} + (C_2 R_1)^{-1} &= a + b \\ (C_1 C_2 R_1 R_2)^{-1} &= ab. \end{aligned} \quad (37)$$

while for Fig. 6, the corresponding relations are

$$\begin{aligned} (C_1 R_1)^{-1} &= K(b - a) \\ (C_1 R_1)^{-1} + (C_2 R_2)^{-1} + (C_1 R_2)^{-1} &= a + b \\ (C_1 C_2 R_1 R_2)^{-1} &= ab. \end{aligned} \quad (38)$$

Equal capacitor or equal resistor designs can now be easily derived by appropriate replacements in the results for the network of Fig. 4.

## V. CONCLUSION

A formula, simpler than the existing one, has been given for the design of the double exponential function  $f(t) = K(e^{-at} - e^{-bt})$ . Also, the problem of synthesizing an RC two-port for generating  $f(t)$  from an impulse or step excitation has been discussed in detail, and several alternative designs and structures have been presented for this purpose.

## REFERENCES

- [1] J. M. Googe, P. D. Ewing and R. A. Hess, "Parameter correlation of impulse shapes using two-port synthesis," *IEEE Trans. Instrum. Meas.*, vol. 41, pp. 54-58, Feb. 1992.

## Producing 180° Out-of-Phase Signals From a Sinusoidal Waveform Input

Hossein Golnabi and Ashkan Ashrafi

**Abstract**—In this paper a simple and novel method for producing two signals with 180° phase difference has been introduced. The phase-balanced signals are produced from a single sinusoidal input signal using two matched op-amps. The phase accuracy of the proposed circuit due to mismatched components is investigated with a two-pole model for the operational amplifiers. With high-precision integrated circuits, the phase error at the amplifier pole frequency (2 MHz) is calculated to be about 0.5° which is a remarkable achievement. The discrete implementation of this scheme is also discussed, and there is a good agreement between the experimental results and the calculated values. The main features of this circuit make it suitable for use in bridge measuring systems and lock-in amplifiers.

## I. INTRODUCTION

In many instrumentation systems, the existence of two 180° out-of-phase sinusoidal signals is essential. These two signals could be used for increasing the sensitivity of bridge circuits such as a Wheatstone bridge, which can be used for measuring an unknown resistor in strain gauge systems [1]. The other great advantage of two well-balanced 180° out-of-phase signals is the ability to eliminate the effects of offset in lock-in-based systems [2]. The accuracy and reliability of such signals are, therefore, very important in the performance of high-precision measuring instruments. At low frequencies, generating such signals is not a complicated task, but for high frequencies the effects of nonideal circuit elements will cause deterioration from the ideal response. Two traditional remedies exist for this problem; the first is to use a transistor pair as a differential amplifier, and the second is to use an operational amplifier in the inverting mode. The former is a well-known method, but it has several limitations such as low permissible input voltage levels, and the requirement of finding

Manuscript received October 24, 1994; revised July 24, 1995.

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Publisher Item Identifier S 0018-9456(96)00860-1.

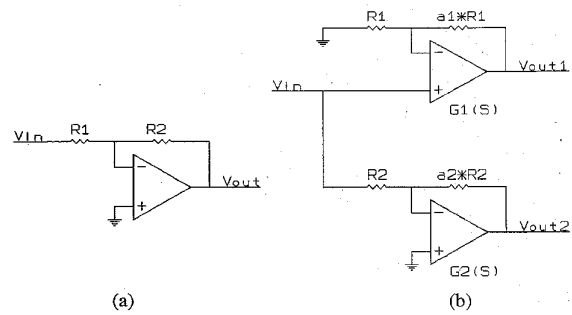


Fig. 1. (a) Conventional op-amp-based inverter, and (b) proposed scheme for producing 180° phase difference.

two highly matched transistors. The main problem with operational amplifiers is low gain-bandwidth product (GBWP). In this paper we model the amplifier performance and minimize phase errors by using matched op-amps.

## II. THEORETICAL ANALYSIS

To develop the theory for our two-matched-op-amp model, we start with a simple first-order transfer function for a conventional op-amp-based inverter for producing two 180° out-of-phase signals (Fig. 1(a)). This method has, however, major limitations at high frequencies. We can write down the overall input-output characteristic of such a simple inverter as

$$\frac{V_o}{V_{in}} = \frac{-R_2 B / R_1}{(R_2 / R_1 + 1)s + B} \quad (1)$$

where  $B$  is the GBWP of the op-amp. As it is apparent in (1), the transfer function of the inverter has a pole at  $\omega = B / (R_2 / R_1 + 1)$ . This pole not only causes a decrease in the gain at the frequencies above it, but also degrades the output phase characteristic at frequencies much below it. To reduce this effect several methods have been tried.

One of the most important methods is to use active feedback with matched operational amplifiers [3], and another approach is to use composite operational amplifiers [4]. These methods are useful in situations where we need to produce a 180° out-of-phase signal from a signal that must maintain its phase integrity and can not be manipulated by any means. However, some situations exist where these two signals are fed into the front-end of the system, so the initial phase of the feeding signal is not important, and the only important factor is the phase difference between these two signals. For this purpose, the use of two matched op-amps (where one of them inverts the input signal, with a residual phase error caused by the nonideal op-amp characteristics, and the other produces a signal with a similar residual phase error) seems to be useful.

Fig. 1(b) shows a simple proposed circuit for producing two 180° out-of-phase signals. As mentioned before, the phase difference between  $v_{o1}$  and  $v_{o2}$  is just equal to 180°, if  $a_1 = a_2$  and the two op-amps match exactly. The two-pole transfer function for each op-amp can be written as

$$G(s) = \frac{B\alpha}{s(s + \alpha)} \quad (2)$$

where  $\alpha$  is the second pole. We can then write down the transfer functions of these two amplifiers as

$$H_1(s) = \frac{(a_1 + 1) \frac{B_1 \alpha_1}{s(s + \alpha_1)}}{\frac{B_1 \alpha_1}{s(s + \alpha_1)} + (a_1 + 1)} \quad (3)$$