Novel Parameter Estimation of Double Exponential Pulse (EMP, UWB) by Statistical Means

Congguang Mao and Hui Zhou

***Abstract*—High-power electromagnetic environments, such as the high-altitude electromagnetic pulse (HEMP) and the ultrawide-band (UWB) pulses, pose dangerous threats to electronic systems. Such pulse shapes are often described physically by the characteristic parameters: the rise time t_r, pulse width t_{fwhm}, and maximum electric field strength $E_{\max}$, and mathematically by the double exponential function with characteristic parameters α, β, k, and E_0. In practice, it is very necessary to transform the two groups of parameters into each other. In this paper, a novel relationship between the two groups of parameters is established by statistical means. This method utilizes only four assistant variables to realize the transform, and the overall estimation error is less than 2.0%.**

***Index Terms*—Double exponential function, electromagnetic pulse (EMP), high-altitude electromagnetic pulse (HEMP), numerical calculation, parameter estimation, ultrawide band (UWB).**

I. Introduction

High-power electromagnetic environments, such as the high-altitude electromagnetic pulse (HEMP) created by nuclear bursts and the ultrawide-band (UWB) pulses, pose dangerous threats to electronic systems. Such pulse shapes are often described physically by the characteristic parameters: the rise time t_r, pulse width t_{fwhm}, and maximum electric field strength $E_{\max}$, and mathematically by the double exponential function with characteristic parameters α, β, k, and E_0. In practical experiments and numerical simulations about the electromagnetic pulses (EMPs), it is very necessary to transform the two groups of parameters into each other. A good nature of the double exponential function is found by numerical calculations, which deduces a parameter estimation method [1]. However, the method is too periphrastic, and there is no proper error analysis. The least squares and Nelder–Mead algorithms are also applied to estimate the physical parameters from the mathematical ones [2], but the equations cannot realize the inverse transform. The main idea of this paper is to explore the intrinsic property between the two groups of parameters, based on which a simple and effective relationship is established. Finally, the estimation errors are analyzed and presented definitely.

II. Double Exponential Pulse Shape

The popular double exponential shape is given by (1) [3] as

$$E(t) = E_0 k(e^{-\alpha t} - e^{-\beta t}) \tag{1}$$

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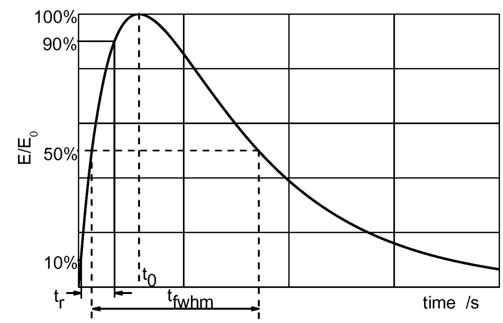

Fig. 1. Double exponential pulse shape and physical parameters.

where E_0 is the maximum of the function, k is a modifying factor, α and β are characteristic parameters, $0 < \alpha < \beta$ to keep the pulse positive polarity, and $t \geq 0$ for t (in seconds[1]) denotes physically the time. The rise time t_r is the time interval between the instants in which the instantaneous amplitude of a pulse first reaches specified lower and upper limits, namely, 10% and 90%, respectively, of the peak pulse amplitude E_0. And the pulse width t_{fwhm} is the time interval between the points on the leading and trailing edges of a pulse at which the instantaneous value is 50% of E_0. The waveform and the definitions of physical parameters are plotted in Fig. 1.

III. Corelation Between (α, β) and (t_r, t_{fwhm})

The maximum value of the pulse can be transformed by [2]

$$E_0 = E_{\max} \qquad k = \frac{1}{(e^{-\alpha t_0} - e^{-\beta t_0})} \tag{2}$$

where the peak time $t_0 = (\ln\,\alpha - \ln\,\beta)/(\alpha - \beta)$.

Given the values of α and β, $t_r = t_3 - t_1$ and $t_{\mathrm{fwhm}} = t_4 - t_2$ (in Fig. 1), where t_1, t_2, t_3, and t_4 are roots of the following nonlinear equation we have

$$E(t) = E_0 k(e^{-\alpha t} - e^{-\beta t}) = cE_0 k(e^{-\alpha t_0} - e^{-\beta t_0}).$$

Namely,

$$e^{-\alpha t} - e^{-\beta t} = c(e^{-\alpha t_0} - e^{-\beta t_0}). \tag{3a}$$

Further,

$$e^{-\alpha t} - e^{-\lambda \alpha t} = c(e^{-\ln \lambda/(\lambda-1)} - e^{-\lambda \ln \lambda/(\lambda-1)}) \tag{3b}$$

where $c = 0.1, 0.5$, or 0.9, and $\lambda = \beta/\alpha$. Equations (3a) and (3b) reveal that the k-factor has nothing to do with t_r or t_{fwhm}, and specified the values of λ and $c\,(0 \leq c \leq 1)$, αt will be a constant. This means that α is inversely proportional with the chosen root

[1]The physical units are omitted in the following text for convenience.

TABLE I
VALUES OF FOUR ASSISTANT VARIABLES AND MAXIMUM ESTIMATION ERRORS

β/α	t_{fwhm}/t_r	A	B	C	D	Maximum Estimation Errors (%)
(1.1,1.15]	(4.29458,4.29882]	0.08483	4.20127	1.17449	1.27480	0.3
[1.15,1.2]	[4.29882,4.30441]	0.11146	4.17066	1.15737	1.29448	0.3
[1.2,1.3]	[4.30441,4.31874]	0.14510	4.13011	1.13370	1.32302	0.3
[1.3,1.4]	[4.31874,4.33720]	0.18200	4.08240	1.10520	1.36005	0.3
[1.4,1.5]	[4.33720,4.35834]	0.21140	4.04108	1.08000	1.39531	0.3
[1.5,1.7]	[4.35834,4.40733]	0.24495	3.99042	1.04720	1.44486	0.3
[1.7,2.0]	[4.40733,4.49058]	0.28387	3.92403	1.00269	1.52093	0.3
[2.0,2.5]	[4.49058,4.65239]	0.32032	3.85067	0.94995	1.62731	0.3
[2.5,3]	[4.65239,4.82468]	0.34484	3.78972	0.90373	1.74249	0.3
[3,4]	[4.82468,5.18725]	0.36245	3.73644	0.85939	1.87732	0.3
[4,6]	[5,6]	0.37545	3.68432	0.81212	2.06921	0.6
[6,10]	[6,7.45]	0.37810	3.66936	0.77381	2.30288	0.7
[10,20]	[7.45,11]	0.36724	3.78565	0.74330	2.61797	0.7
[20,49]	[11,21]	0.34718	4.20629	0.71849	3.13579	0.7
[49,600]	[21,200]	0.32500	5.38116	0.69852	4.27180	2.0

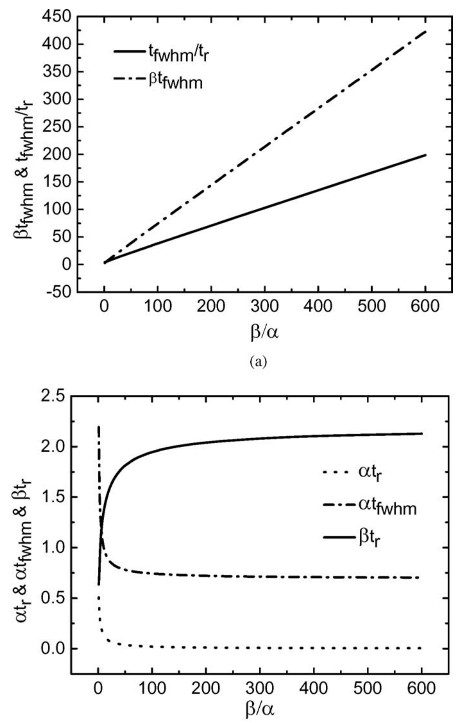

Fig. 2. Variations of combinations of $t_{\rm fwhm}$, t_r, β, and α versus β/α. (a) $t_{\rm fwhm}/t_r$ and $\beta t_{\rm fwhm}$ variations versus β/α. (b) αt_r, $\alpha t_{\rm fwhm}$, and βt_r variations versus β/α.

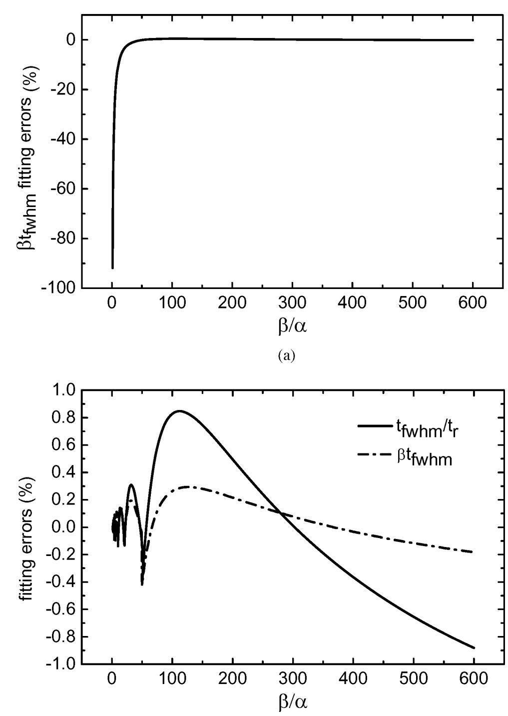

Fig. 3. Fitting errors of $t_{\rm fwhm}/t_r$ and $\beta t_{\rm fwhm}$ versus β/α with straight lines. (a) Fitting errors of $\beta t_{\rm fwhm}$ versus β/α with a single straight line. (b) Fitting errors of $t_{\rm fwhm}/t_r$ and $\beta t_{\rm fwhm}$ versus β/α with subsection straight lines.

t, and then so is α with t_r and $t_{\rm fwhm}$ [1]. Take randomly $\lambda = 15$ as an example with α varying, one can obtain different values of β, t_r, and $t_{\rm fwhm}$, while $\alpha t_r \equiv 0.09877$ and $\alpha t_{\rm fwhm} \equiv 0.91922$. Here, the nonlinear equation (3a) is solved by the bisection method, and the calculation errors are less than 10^{-15} [4].

At sampling points in the intervals $\alpha \in [1 \times 10^8,\ 4 \times 10^8]$ and $\beta \in [5 \times 10^8,\ 600 \times 10^8]$, t_r, $t_{\rm fwhm}$, and some of their products and quotients are calculated. The results are plotted as functions of β/α. It is very interesting that the values of $t_{\rm fwhm}/t_r$ and $\beta t_{\rm fwhm}$ appear as two straight lines [see Fig. 2(a)], while the other products appear as strong curves [see Fig. 2(b)]. So, the correlations between $t_{\rm fwhm}/t_r$, $\beta t_{\rm fwhm}$, and β/α are formulated as the following linear equations system:

$$\frac{t_{\rm fwhm}}{t_r} = A\frac{\beta}{\alpha} + B \tag{4a}$$

$$\beta t_{\rm fwhm} = C\frac{\beta}{\alpha} + D. \tag{4b}$$

The next step is to determine the assistant variables *A*, *B*, *C*, and *D* with the linear fitting method. With $\alpha = 1 \times 10^8$ and changing values $\beta \in (1.1 \times 10^8, 60 \times 10^9]$, different values of

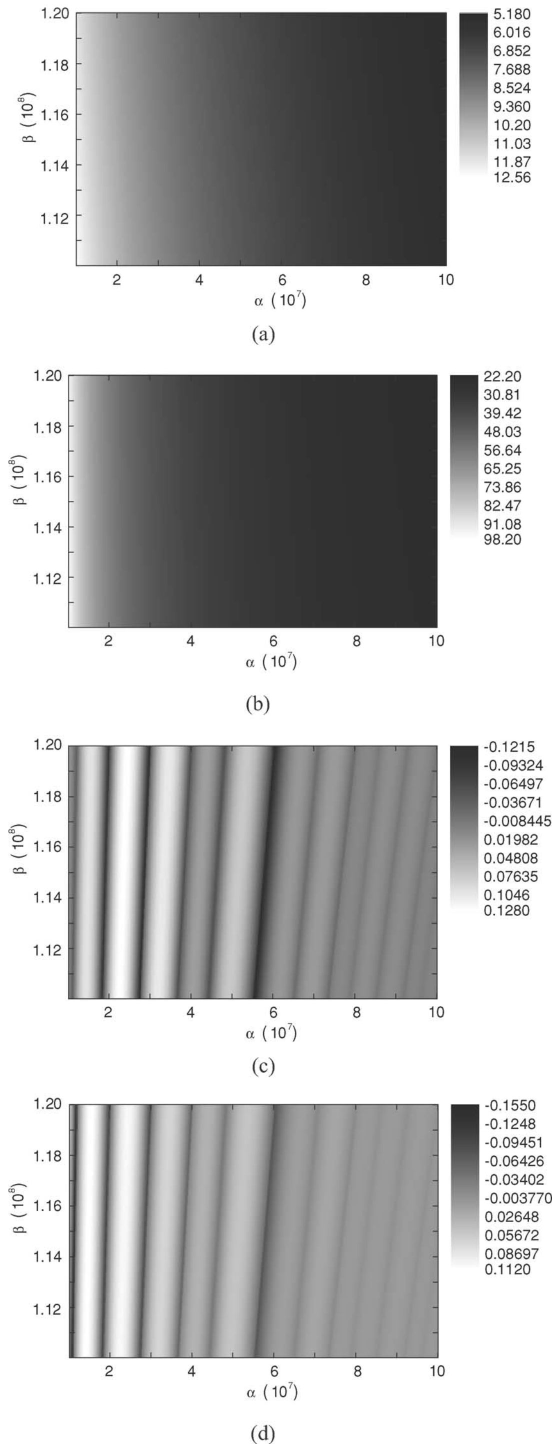

Fig. 4. Values and corresponding estimation errors of t_r and t_{fwhm} in the intervals $\alpha \in [1 \times 10^7,\ 10 \times 10^7]$ and $\beta \in [1.10001 \times 10^8,\ 1.2 \times 10^8]$. (a) Values of t_r (10^{-9}). (b) Values of t_{fwhm} (10^{-9}). (c) Estimation errors of t'_r (in percent). (d) Estimation errors of t'_{fwhm} (in percent).

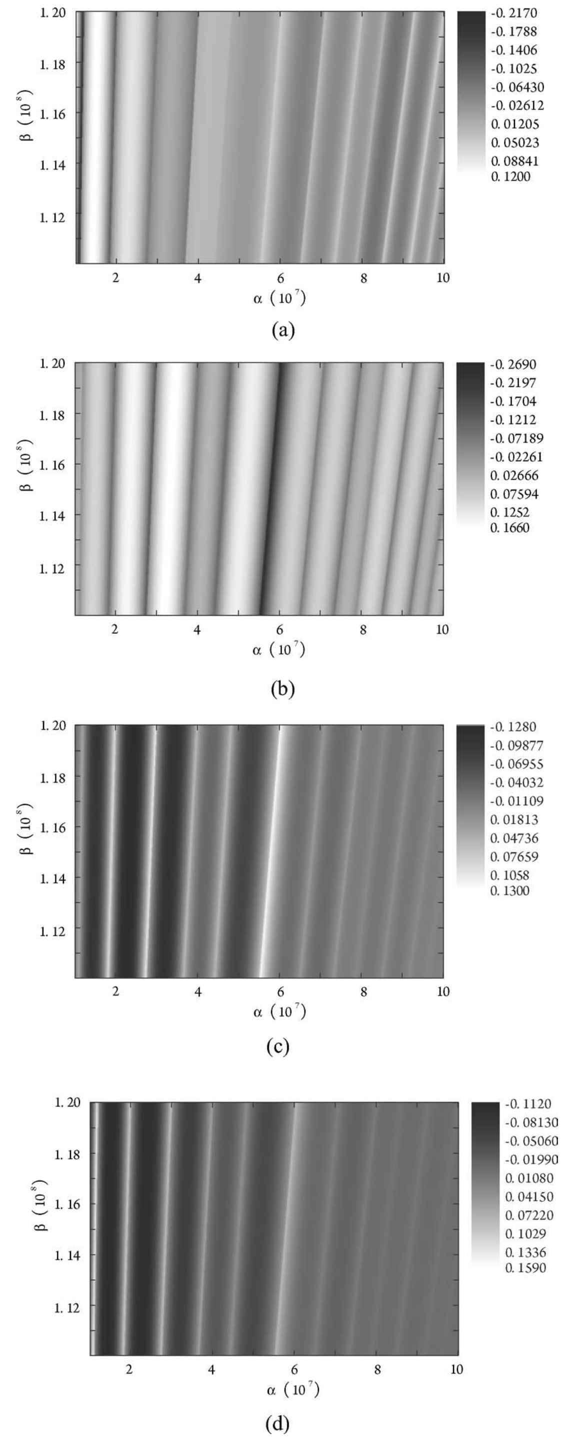

Fig. 5. Estimation errors of α', β', t''_r and t''_{fwhm} in the intervals $\alpha \in [1 \times 10^7,\ 10 \times 10^7]$ and $\beta \in [1.10001 \times 10^8,\ 1.2 \times 10^8]$. (a) Estimation errors of α' (in percent). (b) Estimation errors of β' (in percent). (c) Reproduction errors of t''_r (in percent). (d) Reproduction errors of t''_{fwhm} (in percent).

t_{fwhm}/t_r, βt_{fwhm}, and β/α are calculated. First, a single straight line is adopted to fit the curve βt_{fwhm}, and the fitting errors are calculated by (5), where $X = \beta t_{\mathrm{fwhm}}$

$$\text{error} = \left(1 - \frac{(X)'}{X}\right) \times 100\%. \tag{5}$$

The results show that the errors near the origin are too large. This indicates that it is not a real straight line as it looks, but the more nonlinear the curve appears, the closer it is to the origin [see Fig. 3(a)]. So the curve is divided into several subsections to fit with overall fitting errors <0.5%, and so does (4a) with errors <0.9% [see Fig. 3(b)]. The values of four assistant variables are listed in Table I.

Equations (4a) and (4b) indicate that given the ratio of β/α, $t_{\rm fwhm}$ is directly proportional to t_r and inversely proportional to β, and both the proportion coefficients vary linearly with β/α.

When $\alpha = 1 \times 10^8$ and $\beta = 1.00001 \times 10^8$, the true value of $t_{\rm fwhm}/t_r \approx 4.29100$ and $\beta t_{\rm fwhm} \approx 2.44640$. It implies that β must not be great infinitely, and a pulse shape can be fitted with the double exponential function only when

$$\frac{t_{\rm fwhm}}{t_r} > 4.29100 \tag{6}$$

$$\beta t_{\rm fwhm} > 2.44640. \tag{7}$$

From (4) and Table I,

$$\lim_{\beta \to \alpha} \frac{t_{\rm fwhm}}{t_r} = A + B = 4.28610 \tag{8}$$

$$\lim_{\beta \to \alpha} \beta t_{\rm fwhm} = C + D = 2.44929. \tag{9}$$

The percentage errors are, respectively, -0.114% and 0.118%.

IV. Analysis of Estimation Errors

The next work is to examine the precision of the method. In practice, two cases can be met: one is to obtain t'_r and $t'_{\rm fwhm}$ from α and β, and the other is from t_r and $t_{\rm fwhm}$ to α' and β', where t'_r, $t'_{\rm fwhm}$, α', and $\beta'$are the estimated parameters. The intervals $\alpha \in \left[1 \times 10^7,\ 10 \times 10^7\right]$ and $\beta \in \left[1.10001 \times 10^8,\ 1.2 \times 10^8\right]$ are chosen to validate the reliability, and the precise values of t_r and $t_{\rm fwhm}$ are calculated by the method described earlier.

First, given α and β and from (4)

$$t'_{\rm fwhm} = \frac{C}{\alpha} + \frac{D}{\beta} \tag{10a}$$

$$t'_r = \frac{t'_{\rm fwhm}}{A(\beta/\alpha) + B}. \tag{10b}$$

The estimation errors [by (5), where $X = t_r,\ t_{\rm fwhm}$] of t'_r and $t'_{\rm fwhm}$ are both less than 0.2% [see Fig. 4(c) and (d)], and the value ranges of t_r and $t_{\rm fwhm}$ are plotted in Fig. 4(a) and (b).

Second, given t_r and $t_{\rm fwhm}$ and from (4)

$$\beta' = \frac{C}{A}\left(\frac{1}{t_r} - \frac{B}{t_{\rm fwhm}}\right) + \frac{D}{t_{\rm fwhm}} \tag{11a}$$

$$\alpha' = \frac{A\beta'}{t_{\rm fwhm}/t_r - B}. \tag{11b}$$

The maximum estimation errors [by (5), $X = \alpha,\ \beta$] of α' and β' are both less than 0.3% [see Fig. 5(a) and (b)].

The errors of α' and β' will consequentially degrade the precision of reproductions t''_r and $t''_{\rm fwhm}$. So, based on α' and β', the true values of t''_r and $t''_{\rm fwhm}$ are also calculated and compared with t_r and $t_{\rm fwhm}$. The final errors calculated by (5) are less than 0.2% [see Fig. 5(c) and (d)].

The validation procedures of other intervals are the same as mentioned earlier. The maximum estimation error of the six ones in every interval is denoted in the last column of Table I.

V. Conclusion

In this paper, a novel parameter estimation method is proposed for the double exponential pulse. With this method, the physical and mathematical parameters can be transformed from each other. The estimation errors of t'_r, $t'_{\rm fwhm}$, α', and β' are all less than 2.0%, and the errors of the reproductions t''_r and $t''_{\rm fwhm}$ are less than 1.0%. Furthermore, if the ranges of t_r and $t_{\rm fwhm}$ or α and β are beyond that given earlier, it is believed that the estimations can also be obtained by this idea.

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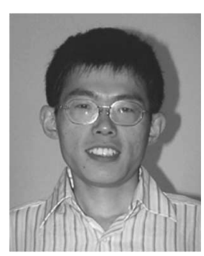

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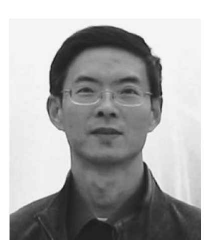

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