Parameter Estimation of Double Exponential Pulses (EMP, UWB) With Least Squares and Nelder Mead Algorithm

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***Abstract*—Transient testpulses like electromagnetic pulse and ultrawide-band are commonly described by rise time t_r, pulse length t_{fwhm} and the maximum amplitude $\hat{E}$. Simulating the effects of varying pulses an analytical description of the pulses is necessary, which is given by a double exponential form with the parameters α, β and E_0. This paper describes the link between the parameters t_r, t_{fwhm} and $\hat{E}$ on the one side and the analytical parameters α, β and E_0 on the other side. It is shown that the Nelder Mead Simplex Algorithm in combination with the least squares method is appropriate to determine an analytical relationship between these parameters. There with extensive analysis of double exponential pulse shapes with far-ranging parameters is possible in a considerably smaller computing time.**

***Index Terms*—Double exponential pulse shape, electromagnetic pulse (EMP), high-power electromagnetics (HPEM), Intentional Electromagnetic Interference (IEMI), least squares, Nelder Mead, parameter, ultrawide-band (UWB).**

I. Introduction

RISKS as a result of upset effects of electronic circuits are ranging from harmless breakdown effects of household appliances to perilous failure effects of medical equipment, culminating in a total collapse of traffic-, communication- and defense-systems of modern developed nations, with fatal consequences for the affected areas [1]. Taken the aspect of electromagnetic terrorism into account, pulses with fast rise times and pulse lengths [electromagnetic pulse (EMP), UWB] are posing a dangerous threat, because new developed pulse generating devices can be built in a very small volume due to the low energy content of the pulse. These pulse shapes can be approximated by a double exponential function [2]. The mathematical parameters of this function are linked with the commonly used parameters rise time t_r and pulse length t_{fwhm} [3] via a nonlinear system of equations. To predict the upset effects of electronic equipment, extensive spectral energy distribution analysis of double exponential pulse shapes are necessary. On this account a direct connection between the mathematical and the commonly used parameters is required.

In this paper, an analytical connection between the mathematical pulse parameters α, β and E_0 with the commonly used parameters rise time t_r, pulse length t_{fwhm} and $\hat{E}$ is determined.

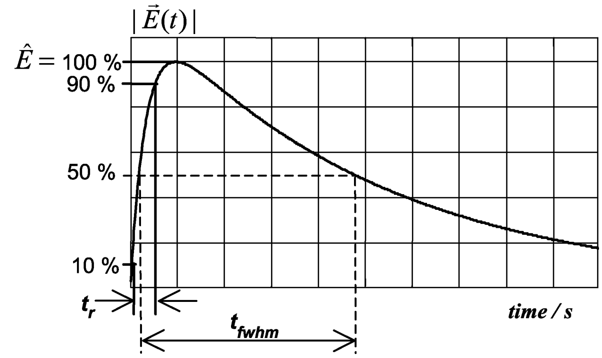

Fig. 1. Double exponential pulse shape.

It is shown that the Nelder Mead Simplex Algorithm [4] in combination with the least squares method [5] is appropriate to determine the relationship between the parameters. In addition a new amplitude factor for double exponential pulse shapes is defined, to realize constant pulse amplitudes with different mathematical pulse parameters.

II. Double Exponential Pulse Shapes

Different mathematical forms to describe EMP or UWB pulse shapes can be found in the literature [2], [3]. Very popular is the following double exponential form (1) with the unit step function $u(t)$:

$$\left|\vec{E}(t)\right| = E_0(e^{-\alpha t} - e^{-\beta t}) \cdot u(t) \qquad (1)$$

With two exponential functions arbitrary EMP or UWB pulse shapes can be realized via the parameters α and β. Commonly used parameters like rise time t_r, pulse length t_{fwhm} or amplitude are not ascertainable directly. Fig. 1 shows a double exponential pulse shape calculated via (1).

To describe commonly used parameters like rise time t_r or pulse length t_{fwhm} different definitions can be found in the literature [4]. In this paper, the rise time t_r is defined as the time period within the pulse is changing from 10% to 90% of the maximum value. The pulse length t_{fwhm} is defined as the time period from the 50% value at the rising edge to the 50% value at the falling edge (Fig. 1).

III. Determination of Amplitude Factor

To create different double exponential pulse shapes via (1) with variable α/β-parameters but a constant amplitude $\hat{E}$ (condition precedent to far-ranging analysis) an additional K-factor is introduced. Without this new factor it is not possible to create

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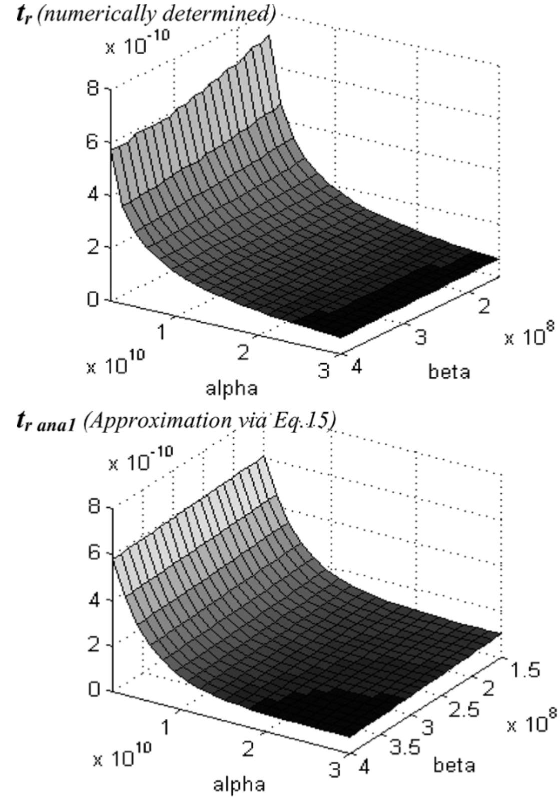

Fig. 2. Numerically determined values of the rise time t_r in comparison with the results of (15).

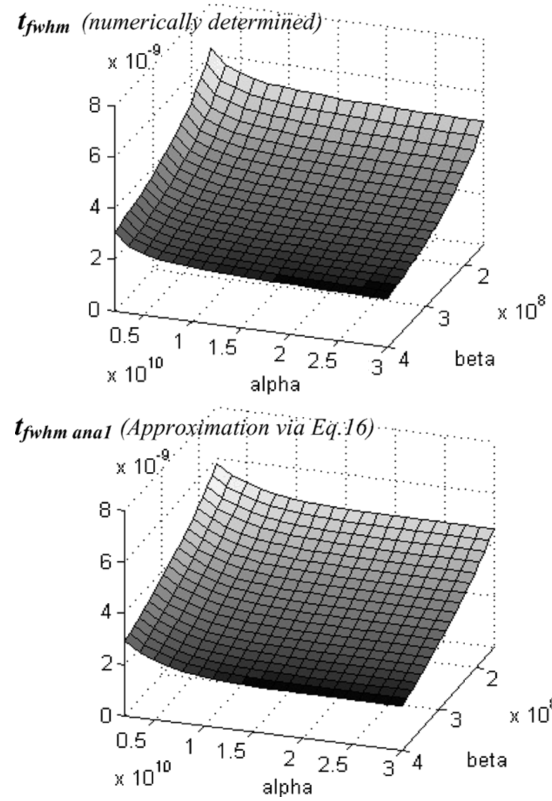

Fig. 3. Numerically determined values of the pulse length t_{fwhm} in comparison with the results of (16).

arbitrary double exponential pulse shapes with different parameters and constant amplitudes accurately and in a small computing time for any analysis.

With (1) follows

$$E(t) = \hat{E} \cdot K(\alpha, \beta) \cdot (e^{-\alpha t} - e^{-\beta t}) \cdot u(t) \tag{2}$$

and the derivative

$$\frac{dE(t)}{dt} = \hat{E} \cdot K(\alpha, \beta) \cdot (-\alpha \cdot e^{-\alpha t} + \beta \cdot e^{-\beta t}) \cdot u(t) \tag{3}$$

At the maximum value of the electrical field strength $\hat{E}$ from (3) follows

$$\frac{dE(t_{\max})}{dt} = \hat{E} \cdot K(\alpha, \beta) \cdot (-\alpha \cdot e^{-\alpha t_{\max}} + \beta \cdot e^{-\beta t_{\max}}) \cdot u(t) = 0 \tag{4}$$

with the time $t_{\max}$ from the beginning of the pulse to the maximum value. After conversion $t_{\max}$ can be calculated via (5)

$$t_{\max} = \frac{\ln(\beta) - \ln(\alpha)}{(\beta - \alpha)}. \tag{5}$$

Insertion of (5) into (6)

$$E(t_{\max}) = \hat{E} \cdot K(\alpha, \beta) \cdot (e^{-\alpha t_{\max}} - e^{-\beta t_{\max}}) \cdot u(t) \tag{6}$$

results in the essential K-factor

$$K(\alpha, \beta) = \left(e^{-\alpha \frac{\ln(\beta) - \ln(\alpha)}{(\beta - \alpha)}} - e^{-\beta \frac{\ln(\beta) - \ln(\alpha)}{(\beta - \alpha)}}\right)^{-1}. \tag{7}$$

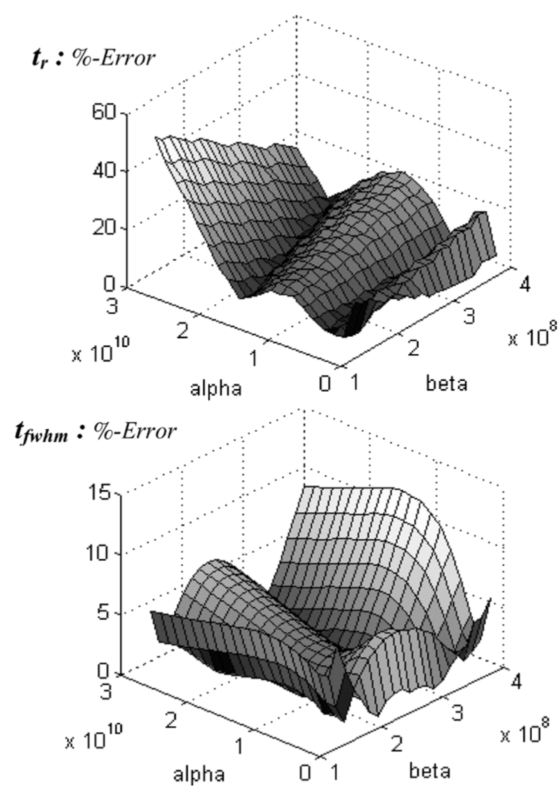

Fig. 4. Percentage error of rise time t_r and pulse length t_{fwhm} determined numerically and via (15) and (16).

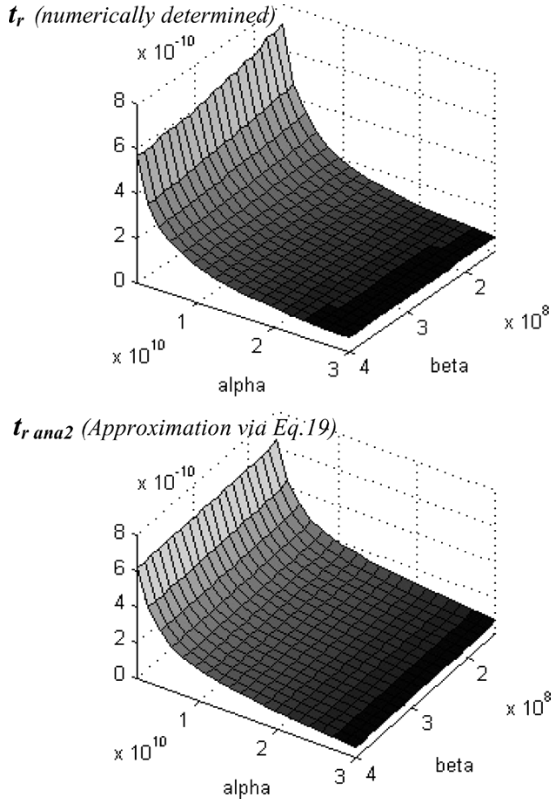

Fig. 5. Numerically determined values of the rise time t_r in comparison with the results of (19).

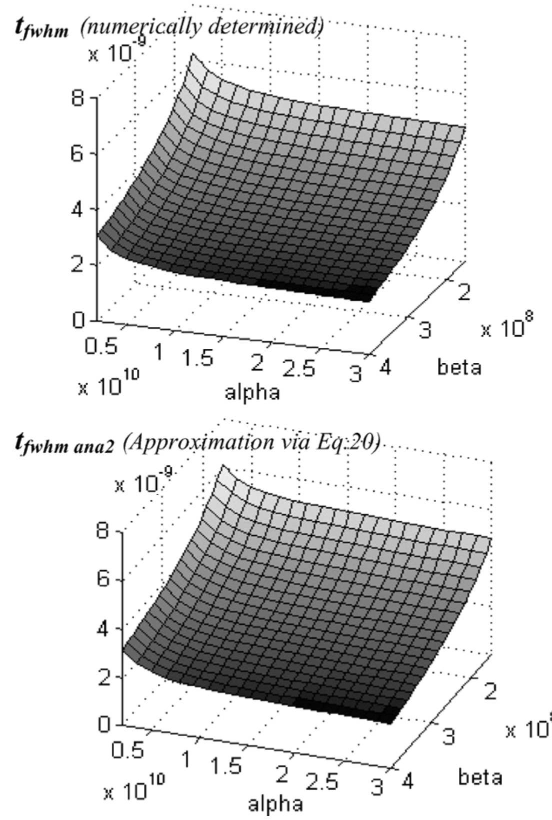

Fig. 6. Numerically determined values of the pulse length t_{fwhm} in comparison with the results of (20).

Therewith, it is possible to create arbitrary double exponential pulse shapes by changing the parameters α and β with constant amplitudes $\hat{E}$. This is the first step to perform extensive analysis of double exponential pulse shapes with far-ranging parameters.

IV. Correlation of Mathematical and Commonly Used Parameters

Very often the rough approximation $t_r \approx 1/\beta$ and $t_{fwhm} \approx 1/\alpha$ can be found in the literature. The results are increasingly poor if the values of the parameters α and β, resp. t_r and t_{fwhm} are becoming similar. In fact the correlation of the mathematical parameters α and β with the commonly used parameters rise time t_r and pulse length t_{fwhm} is given by the following nonlinear system of equations which cannot be solved analytically

$$E(t_{50\%}) = \frac{\hat{E}}{2} = \hat{E}\cdot K(\alpha,\beta)\cdot(e^{-\alpha t_{50\%}} - e^{-\beta t_{50\%}}) \quad (8)$$

$$E(t_{10\%}) = \frac{\hat{E}}{10} = \hat{E}\cdot K(\alpha,\beta)\cdot(e^{-\alpha t_{10\%}} - e^{-\beta t_{10\%}}) \quad (9)$$

$$E(t_{90\%}) = \frac{9\hat{E}}{10} = \hat{E}\cdot K(\alpha,\beta)\cdot(e^{-\alpha t_{90\%}} - e^{-\beta t_{90\%}}). \quad (10)$$

Although a numerical solution is possible, an analytical correlation is required to perform analysis of double exponential pulse shapes with far-ranging parameters in a small computing time. To determine the correlation between α, β, t_r and t_{fwhm}, 400 double exponential pulses with a linear distribution of α and β have been created via (6). Afterwards the parameters rise time t_r and pulse length t_{fwhm} have been determined numerically. Because of the exponential character of the nonlinear system of [(8)–(10)] and the desire for a simple mathematical expression, a sum of two exponential functions has been chosen to approximate t_r and t_{fwhm}.

The substitution

$$t_{\text{rana1}} = X_1 e^{\alpha X_2} + X_3 e^{\beta X_4} \quad (11)$$

$$t_{fwhm\ \text{ana1}} = X_5 e^{\alpha X_6} + X_7 e^{\beta X_8} \quad (12)$$

is leading with the criterion to minimize the following sum (least squares method [5])

$$\min\left(\sum\left(t_r - (X_1 e^{\alpha X_2} + X_3 e^{\beta X_4})\right)^2\right) \quad (13)$$

$$\min\left(\sum\left(t_{fwhm} - (X_5 e^{\alpha X_6} + X_7 e^{\beta X_8})\right)^2\right) \quad (14)$$

by application of the Nelder Mead Algorithm [4] in the area $2\cdot 10^9 < \alpha < 30\cdot 10^9$ and $150\cdot 10^6 < \beta < 400\cdot 10^6$ to the approximation

$$t_{\text{rana1}} = 1*10^{-10} e^{\alpha\cdot(-1,2*10^{-11})} + 29*10^{-8} e^{\beta\cdot(-2,7*10^{-8})} \quad (15)$$

$$t_{fwhm\,\text{ana1}} = 4.95*10^{-9} e^{\alpha\cdot(-3.5*10^{-11})} + 72*10^{-8} e^{\beta\cdot(-2,7*10^{-8})}. \quad (16)$$

Fig. 2 and 3 are showing the numerically determined values of t_r and t_{fwhm} in comparison with the calculated values via (15) and (16).

In principle the appearance and characteristics are similar. However the percentage error in the worst case is about 45% for the rise time t_r and about 11% for the pulse length t_{fwhm}.

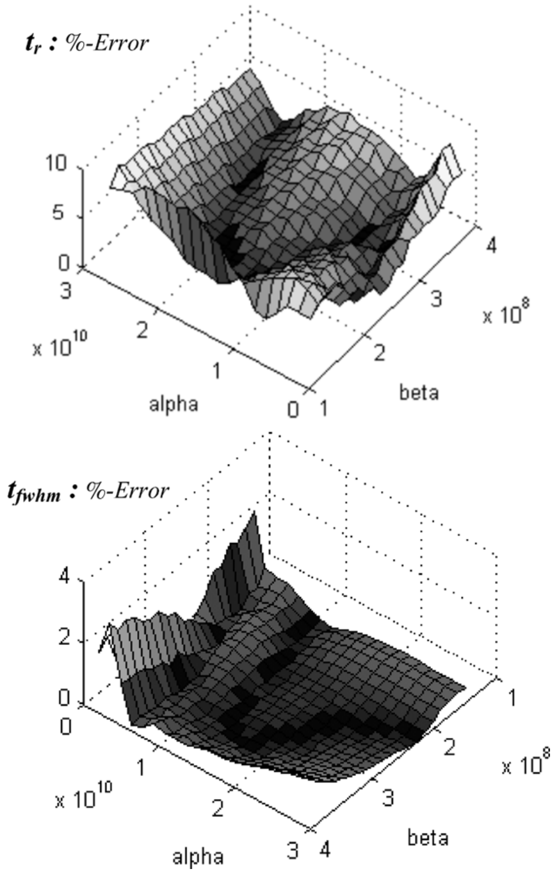

Fig. 7. Percentage error of rise time t_r and pulse length t_{fwhm} determined numerically and via (19) and (20).

Fig. 4 shows the percentage error of the rise time t_r and the pulse length t_{fwhm}.

Hence, to improve the approximation, a sum of four exponential functions is chosen

$$t_{\text{rana2}} = X_1 e^{\alpha X_2} + X_3 e^{\beta X_4} + X_5 e^{\alpha X_6} + X_7 e^{\beta X_8} \tag{17}$$

$$t_{fwhm\ \text{ana2}} = X_9 e^{\alpha X_{10}} + X_{11} e^{\beta X_{12}} + X_{13} e^{\alpha X_{14}} + X_{15} e^{\beta X_{16}}. \tag{18}$$

Application of the least squares method [5] and Nelder Mead Algorithm [4] in the area $2 \cdot 10^9 < \alpha < 30 \cdot 10^9$ and $150 \cdot 10^6 < \beta < 400 \cdot 10^6$ leads to the approximation

$$t_{\text{rana2}} = 4 * 10^{-6} e^{\alpha \cdot (-3{,}3*10^{-6})} + 56 * 10^{-10} e^{\beta \cdot (5{,}6*10^{-9})} + 2{,}9 * 10^{-8} e^{\alpha \cdot (-3{,}7*10^{-9})} + 66 * 10^{-7} e^{\beta \cdot (-5*10^{-8})} \tag{19}$$

$$t_{fwhm\ \text{ana2}} = 2 * 10^{-9} e^{\alpha \cdot (-1{,}3*10^{-10})} + 16 * 10^{-6} e^{\beta \cdot (-5*10^{-8})} + 33 * 10^{-9} e^{\alpha \cdot (-2{,}1*10^{-11})} + 14 * 10^{-9} e^{\beta \cdot (-7*10^{-9})}. \tag{20}$$

Figs. 5 and 6 are showing the numerically determined values of t_r and t_{fwhm} in comparison with the calculated values via (19) and (20). The results of the approximations are much better than the results of (15) and (16). The percentage error in the worst case is about 7% for the rise time t_r and about 3% for the pulse length t_{fwhm}. Fig. 7 shows the percentage error of the rise time t_r and the pulse length t_{fwhm}.

V. Conclusion

It is shown that the Nelder Mead Simplex Algorithm [4] in combination with the least squares method [5] is appropriate to determine the relationship between the mathematical parameters and the commonly used parameters rise time t_r and pulse length t_{fwhm} of double exponential pulse shapes. With the correlation determined in this paper, it is possible to perform extensive analysis of spectral energy distributions of double exponential pulse shapes with far-ranging parameters in a considerably smaller computing time, because no numerical calculation of the parameters rise time t_r and pulse length t_{fwhm} is necessary. In addition a new amplitude factor for double exponential pulse shapes is defined, to realize constant pulse amplitudes with different mathematical pulse parameters.

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