# Double-Exponential Expression of Lightning Current Waveforms

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**Abstract:**

A lightning waveform is usually defined by the rise time $t_f$, pulse length $t_p$ and peak value $I$. In order to represent lightning current waveforms in an analytic manner, the double-exponential formula is employed to fit with the lightning current waveforms. This paper describes the relationship between the waveform parameters $t_f$, $t_p$, $I$ and the formula constants $\alpha, \beta$ and $A$. An effective algorithm is proposed to calculate $\alpha$, $\beta$ and $A$. The algorithm gives a reasonable expression for $\alpha, \beta$ and $A$ from the data points of lightning current waveforms. Numerical examples are derived from the algorithm for a few types of lightning current waveforms that are commonly used in lightning protection design. A better agreement appears between calculated and standard waveforms.

**Index Terms:** lightning current, double -exponential function, lightning protection, impulse waveform.

## I. Introduction

Mathematical expression of lightning current waveform is important to lightning protection design. In previous research there were a few channel-base current functions proposed by direct measurements at the top of the masts [1]. For the lightning protection design initiated, the previous lightning current analytical functions are so simple that they are difficult to be used to investigate the direct lightning stroke to different earthed objects such as tall towers and buildings. These features were not always convenient for the study of the lightning protection design. The main problem was the elimination of the discontinuity of the current derivative at the time onset. Two different basic methods of lightning current parameter calculation are usually applied [2]: one is direct measurement estimation method and the other is an iterative method. Both are used to check the validity of the appropriate return stroke model. Recently an adequate channel-base current function has been presented [3], which can yield a good approximation to the observed impulse waveforms of lightning currents by a double-exponential expression. The formula constants $\alpha, \beta$ and $A$ of this expression are associated with the commonly used waveform parameters i.e. the rise time $t_f$ and pulse length $t_p$ and current peak $I$. The need exists, however, for calculation the formula constants $\alpha, \beta$ and $A$ in the terms of these waveform parameters.

This paper presents an effective algorithm to determine the formula constants $\alpha, \beta$ and $A$ for the double-exponential expression of lightning current waveforms. The algorithm develops a direct relationship between the formula constants $\alpha, \beta, A$ and the waveform parameters $t_f$, $t_p$, $I$ and has capability to describe the nonlinear feature of the impulse waveforms in a more accurate manner. Numerical examples are also given to confirm the validity of the algorithm.

## Ⅱ. Double-exponential Formula for Lightning Currents

In lightning protection studies, different mathematical expressions have been used to describe lightning current waveforms. The most popular one of them is following double-exponential expression

$$i(t) = I(e^{-\alpha t} - e^{-\beta t}) \qquad (1)$$

where $\alpha$ and $\beta$ are the formula constants and remain to be determined for the impulse waveforms of lightning currents. Fig. 1 shows a double－exponential lightning

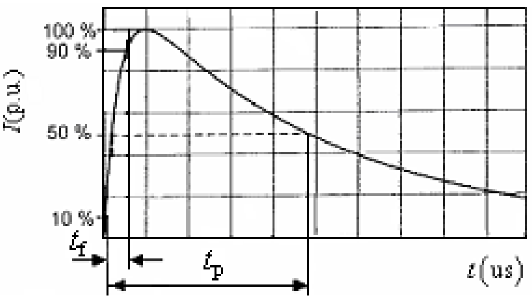



Fig. 1 Double exponential impulse waveform

current waveform. The rise time $t_f$ and the pulse length $t_p$ are also illustrated here. According to the waveform parameters $t_f$, $t_p$ and $I$, the formula constants $\alpha$ and $\beta$ need to be determined for obtaining an analytic expression of the lightning current waveform.

## Ⅲ. Expression for *A*-factor

In the previous double-exponential formula, the constants $\alpha$ and $\beta$ are usually difficult to fit the impulse waveforms of lightning currents. For this reason, an additional $A$-factor must be introduced to create different impulse waveforms in a more accurate way. $A$-factor is important to create the double-exponential impulse waveforms with different parameters and amplitudes.

With respective to eqn.（1）, the modified formula for lightning current is written as

$$i(t) = AI(e^{-\alpha t} - e^{-\beta t}) \qquad (2)$$

and its derivative is

$$\frac{di(t)}{dt} = AI\left(-\alpha e^{-\alpha t} + \beta e^{-\beta t}\right) \qquad (3)$$

At the peak value $I$, eqn. (3) should satisfy the following condition

$$\frac{di(t_{\max})}{dt} = AI\left(-\alpha e^{-\alpha t_{\max}} + \beta e^{-\beta t_{\max}}\right) = 0 \qquad (4)$$

where $t_{\max}$ is the time from the beginning of the current impulse to its peak value. From eqn. (3), the time $t_{\max}$ can be expressed as

$$t_{\max} = \frac{\ln(\beta) - \ln(\alpha)}{(\beta - \alpha)} \qquad (5)$$

Substituting eqn.(5) into eqn.(2) yields

$$i(t_{\max}) = AI\left(e^{-\alpha t_{\max}} - e^{-\beta t_{\max}}\right) \qquad (6)$$

As a result, the $A$-factor can be given as

$$A(\alpha,\beta) = \left(e^{-\alpha\frac{\ln(\beta)-\ln(\alpha)}{(\beta-\alpha)}} - e^{-\beta\frac{\ln(\beta)-\ln(\alpha)}{(\beta-\alpha)}}\right)^{-1} \qquad (7)$$

According to eqs. (2)～(7), it is possible to create arbitrary double exponential expression for lightning current waveform by changing $\alpha$, $\beta$ and $A$.

## Ⅳ. Determination of the Formula Constants

The following rough approximation is given frequently in the literature [3].

$$t_f \approx \frac{1}{\beta} \qquad (8)$$

$$t_p \approx \frac{1}{\alpha} \qquad (9)$$

However, if the values of $\alpha$, and $\beta$, respective to $t_f$ and $t_p$ are becoming similar, the results are increasingly poor. The reason is that the correlation of the mathematical constants $\alpha$ and $\beta$ with the rise time $t_f$ and pulse length $t_p$ is non-linear relationship. This relationship is described by the following equations.

$$I\left(t_{50\%}\right)=\frac{I}{2}=AI\left(e^{-\alpha\, t_{50\%}}-e^{-\beta\, t_{50\%}}\right) \qquad (10)$$

$$I\left(t_{30\%}\right)=\frac{3I}{10}=AI\left(e^{-\alpha\, t_{30\%}}-e^{-\beta\, t_{30\%}}\right) \qquad (11)$$

$$I\left(t_{10\%}\right)=\frac{I}{10}=AI\left(e^{-\alpha\, t_{10\%}}-e^{-\beta\, t_{10\%}}\right) \qquad (12)$$

$$I\left(t_{90\%}\right)=\frac{9I}{10}=AI\left(e^{-\alpha\, t_{90\%}}-e^{-\beta\, t_{90\%}}\right) \qquad (13)$$

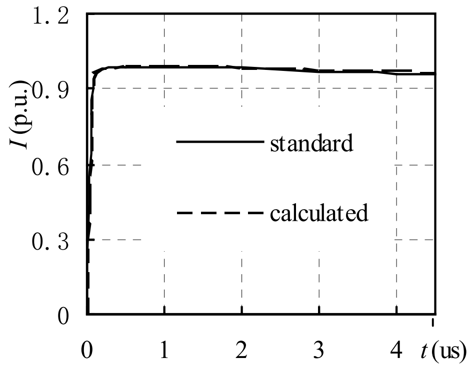
Based on eqs.(10) ~ (13), the formula constants $\alpha$, $\beta$ and $A$ can be determined numerically by a trail and error procedure. Then, the double-exponential expression for lightning current waveforms can be obtained from the formula constants $\alpha$, $\beta$ and $A$. We chose four standard lightning current waveforms and then calculate their correlated constants $\alpha$, $\beta$ and $A$. Table 1 gives the numerically determined values of the four standard lightning current waveforms which are taken from Chinese national standards [6,7].

Tab.1 Calculated formula constants

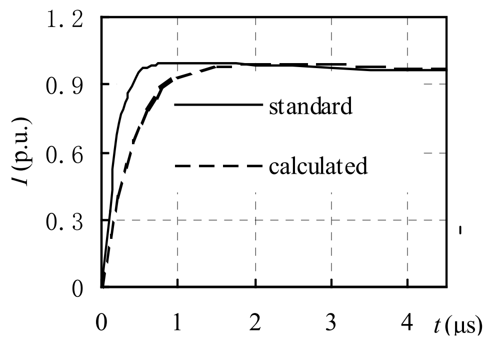
| Parameter | $A$ | $\alpha$（s） | $\beta$（s） |
|---|---|---|---|
| 0.25/100 μs | 1.002 | $7.00\times10^{3}$ | $3.40\times10^{7}$ |
| 1.2/50 μs | 1.037 | $1.47\times10^{4}$ | $2.47\times10^{6}$ |
| 2.6/50 μs | 1.058 | $1.50\times10^{4}$ | $1.86\times10^{6}$ |
| 10/350 μs | 1.025 | $2.05\times10^{3}$ | $5.64\times10^{5}$ |

## Ⅴ. Numerical Examples

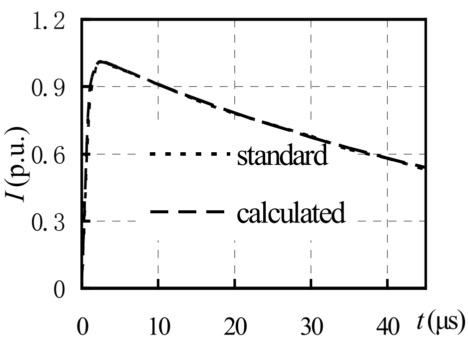
In order to check the validity of the algorithm proposed in this paper, typical numerical examples are delivered here. The waveform parameters $t_f$ and $t_p$ are taken from the standard data related to lightning protection design [6, 7]. The calculated waveforms are illustrated in Fig. 2 (a)~(d), where the corresponding standard waveforms are simultaneously given for comparison. As seen from Fig. 2, a better agreement appears between calculated and standard waveforms.



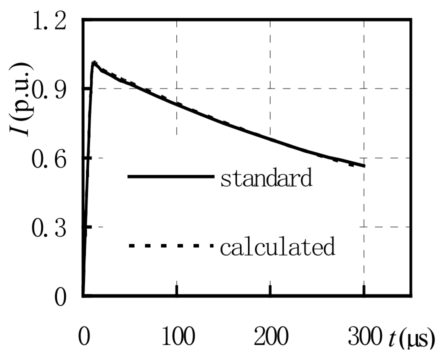

(a) 0.25/100 μs




(b) 1.2/50 μs




(c) 2.6/50 μs




(d) 10/350 μs

Fig. 2 Numerical examples

## Ⅵ. Conclusions

A double-exponential formula is employed in this paper for representing the impulse waveforms of lightning currents. The formula constants associated with the double-exponential function are determined by an effective algorithm. The algorithm gives a direct relationship between the formula constants and the waveform parameters and is convenient for calculating the formula constants. The calculated waveforms of lightning currents are compared with the standard ones and a better agreement is shown in between them, which confirms the validity of the algorithm.

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