Estimation of the Mathematical Parameters of Double-Exponential Pulses Using the Nelder–Mead Algorithm

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and Ralf Vick

***Abstract*—Transient pulses for electromagnetic compatibility problems, such as the high-altitude electromagnetic pulse and ultrawideband pulses, are often described by a double-exponential pulse. Such a pulse shape is specified physically by the three characteristic parameters rise time t_r, pulsewidth t_{fwhm} (full-width at half-maximum), and maximum amplitude $E_{\max}$. The mathematical description is a double-exponential function with the parameters α, β, and E_0. In practice, it is often necessary to transform the two groups of parameters into each other. This paper shows a novel relationship between the physical parameters t_r and t_{fwhm} on the one hand and the mathematical parameters α and β on the other. It is shown that the least-squares method in combination with the Nelder–Mead simplex algorithm is appropriate to determine an approximate closed-form formula between these parameters. Therefore, the extensive analysis of double-exponential pulses is possible in a considerably shorter computation time. The overall approximation error is less than 3.8%.**

***Index Terms*—Double-exponential pulse, high-altitude electromagnetic pulse, Nelder–Mead, parameter estimation, pulsewidth, rise time, ultrawideband pulse.**

I. Introduction

A brief introduction about double-exponential pulses and their importance for the electromagnetic compatibility is summarized in [1]. For practical application, it is often necessary to transform between the physical parameters rise time t_r, pulsewidth t_{fwhm} (full-width at half-maximum), maximum field strength $E_{\max}$, and the mathematical parameters α, β, and E_0.

A very effective relationship that uses only four assistant variables is established in [2]. Since these four variables have different values for different ratios of β/α to give a reasonable overall fitting error, this is not very straightforward. With the help of the least-squares method and the Nelder–Mead algorithm, an estimation of the physical parameters from the mathematical ones is applied in [3]. Unfortunately, these equations cannot be used for the inverse transformation. The main idea of this paper is to give closed-form formulas for the mathematical parameters of the double-exponential pulse from the physical ones with a method that is based on the algorithm presented in [3]. Finally, the estimation errors are analyzed and shown.

II. Double-Exponential Pulse Shapes

The double-exponential shape is given in [1] as follows:

$$E(t) = E_0 k \left(e^{-\alpha t} - e^{-\beta t}\right) h(t) \tag{1}$$

where E_0 is the amplitude, α and β are the characteristic mathematical parameters, and $h(t)$ is the unit-step function. The amplitude factor k is necessary to create different double-exponential pulse shapes with variable parameters, but a constant amplitude. Therefore, k is given in [3] by

$$k = k(\alpha,\beta) = \left[e^{-\alpha \frac{\ln(\alpha/\beta)}{\alpha-\beta}} - e^{-\beta \frac{\ln(\alpha/\beta)}{\alpha-\beta}}\right]^{-1}. \tag{2}$$

With the additional k-factor, the maximum amplitude can be converted by $E_{\max} = E_0$.

III. Numerical Solution for the Correlation Between (α,β) and (t_r, t_{fwhm})

Nevertheless, the physical and mathematical parameters have to be transformed by solving a nonlinear system of equations, which cannot be solved analytically. As stated in [2], for an effective numerical solution, it is convenient to use the double-exponential function not as a function of β and α, but as the ratio of both

$$\lambda = \frac{\beta}{\alpha}. \tag{3}$$

For a pulse with positive polarity, $\alpha < \beta$ and $\lambda > 1$. In the same manner, the physical parameters should not be the pulsewidth t_{fwhm} and rise time t_r, but also the ratio of both

$$\mu = \frac{t_{\mathrm{fwhm}}}{t_r}. \tag{4}$$

It has been shown in [2] that $\mu > 4.291$ for all double-exponential pulses. Using only the ratios has the following advantage: if two pulses have the same ratio λ, they will also have the same μ. If the parameters α and β of one pulse are larger than α and β of another pulse with the same λ by a determined factor, then t_r and t_{fwhm} will be lower by that factor and *vice versa*. One can also say that (α,β) and (t_r, t_{fwhm}) of two pulses are reciprocally proportional to each other, if these two pulses share the same λ and μ. The advantage of this simplification is that the nonlinear optimization has to be performed only in one dimension and not in two, as in [3].

A. Determination of (t_r, t_{fwhm}) From (α,β)

The rise time t_r can be calculated by $t_r = t_{90\%} - t_{10\%}$. The instants of time $t_{10\%}$ and $t_{90\%}$ can be found by an iterative approach

$$t_{10\%,i+1} = -\frac{\log\left(e^{-\alpha t_{10\%,i}} - (0.1/k)\right)}{\beta} \tag{5}$$

$$t_{90\%,i+1} = -\frac{\log\left(e^{-\alpha t_{90\%,i}} - (0.9/k)\right)}{\beta} \tag{6}$$

with the starting parameters

$$t_{10\%,1} = -\frac{\log(1-(0.1/k))}{\beta} \quad t_{90\%,1} = -\frac{\log(1-(0.9/k))}{\beta}. \tag{7}$$

The iteration ends if the relative error between $t_{r,i}$ and $t_{r,i+1}$ is smaller than a preset accuracy. This procedure is more precise, but also more time-consuming than using the closed-form formulas given in [3]. The normalized rise time for different values of $1 < \lambda \le 200$ is shown in Fig. 1. The pulsewidth t_{fwhm} can be calculated with a similar approach. The normalized pulsewidth is also plotted in Fig. 1.

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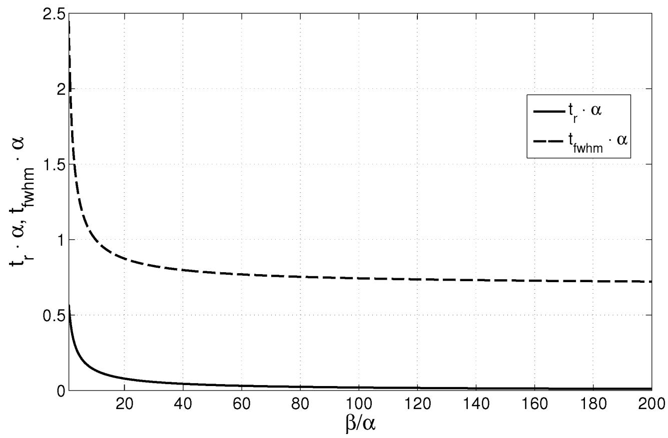

Fig. 1. Rise time t_r and pulsewidth t_{fwhm} normalized to α for different ratios of β to α.

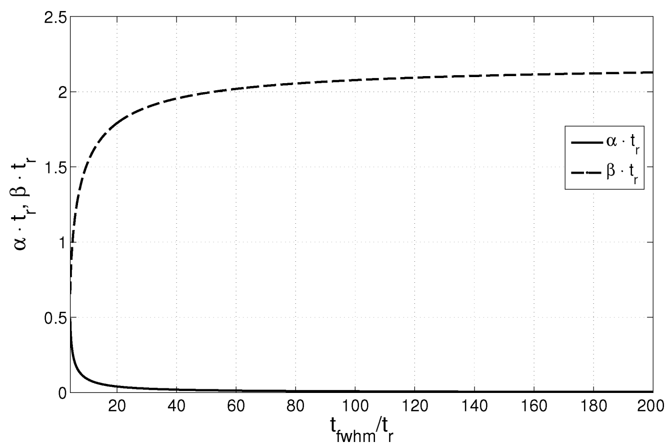

Fig. 2. Parameters α and β normalized to t_r for different ratios of t_{fwhm} to t_r.

B. Determination of (α, β) From (t_r, t_{fwhm})

The determination of α and β from t_r and t_{fwhm} is much more difficult. For a first start, the very rough approximations

$$\alpha = \frac{1}{t_{\mathrm{fwhm}}} \qquad \text{and} \qquad \beta = \frac{1}{t_r} \tag{8}$$

are used. Then, a search algorithm is started. The parameters α and β are slightly changed in a specific interval upward and downward in each step. The pulsewidths and rise times are calculated for each step, as described in Section III-A. These calculated physical parameters are compared to the target parameters for each changed variant of α and β. At the end of each step, the best fitting variant is chosen as the start for the next step. If the best fitting variant is the same as the previous step, the specific interval of change is decreased by half. Therefore, the search is quite fast at the beginning and becomes more and more precise near the end. This is done until the desired pulse parameters t_r and t_{fwhm}, and the calculated parameters have a relative error that is smaller than the preset accuracy. This procedure is quite simple, but very time-consuming. Even so, it offers the possibility to transform the physical parameters of the double-exponential pulse to the mathematical ones. The normalized parameters α and β are plotted for different ratios μ in Fig. 2. The calculation error is only a matter of time and can be as low as 10^{-15}.

TABLE I
PARAMETERS OF THE APPROXIMATION OF α

Parameter	Value
X_1	0.842 411
X_2	1.660 852
X_3	8.887 335
X_4	0.624 129

TABLE II
PARAMETERS OF THE APPROXIMATION OF β

Parameter	Value
Y_1	2.167 370
Y_2	0.360 349
Y_3	0.013 289
Y_4	1.358 701
Y_5	0.137 057
Y_6	122.802 861
Y_7	1.314 850

IV. APPROXIMATION WITH CLOSED-FORM FORMULAS

Because of the hyperbolic character of $\alpha \cdot t_r$ in Fig. 2, a modified hyperbolic function was chosen to approximate the exact value of α

$$\alpha_{\mathrm{approx}} = \frac{1}{t_r} \cdot \frac{X_1}{\left((t_{\mathrm{fwhm}}/t_r)^{X_2} - X_3 \right)^{X_4}}. \tag{9}$$

$X_1, \ldots, X_4$ are the dimensionless parameters of the approximation model. Due to the exponential character of $\beta \cdot t_r$, which is shown in Fig. 2, and the desire for a simple mathematical expression, a sum of three exponential functions was selected to approximate the precise magnitude of β

$$\beta_{\mathrm{approx}} = \frac{1}{t_r} \cdot \Bigg[Y_1 - Y_2 \cdot e^{-\frac{t_{\mathrm{fwhm}}}{t_r} \cdot Y_3} - Y_4 \cdot e^{-\frac{t_{\mathrm{fwhm}}}{t_r} \cdot Y_5} - Y_6 \cdot e^{-\frac{t_{\mathrm{fwhm}}}{t_r} \cdot Y_7} \Bigg]. \tag{10}$$

This approximation model has seven dimensionless parameters $Y_1, \ldots, Y_7$. Using a sum of more exponential functions would be more precise, but would also require more parameters. The closed-form approximation formulas are used with the criterion to minimize the following sums in a specific region $\mu \in [\mu_{\min}, \mu_{\max}]$:

$$\left[\sum_{\mu_{\min}}^{\mu_{\max}} (\alpha - \alpha_{\mathrm{approx}})^2 \right] \xrightarrow{X_1, \ldots, X_4} \min \tag{11}$$

$$\left[\sum_{\mu_{\min}}^{\mu_{\max}} (\beta - \beta_{\mathrm{approx}})^2 \right] \xrightarrow{X_1, \ldots, X_4} \min. \tag{12}$$

This is known as the least-squares method [4]. By applying of the Nelder–Mead algorithm [5] in the region $< \mu \leq 1000$, the parameters shown in Tables I and II for the approximation of the mathematical parameters α and β from the physical ones t_r and t_{fwhm} can be derived.

A comparison between the exact numerical and approximated analytical solution for α and β can be found in Figs. 3 and 4. Despite these plots being logarithmically scaled in both axis directions, almost no differences between the exact and the approximated curve can be found.

V. ANALYSIS OF ESTIMATION ERRORS

The relative errors are used to examine the precision of the method. The estimation errors in (9) and (10) are less than 3.8% and 3.2%,

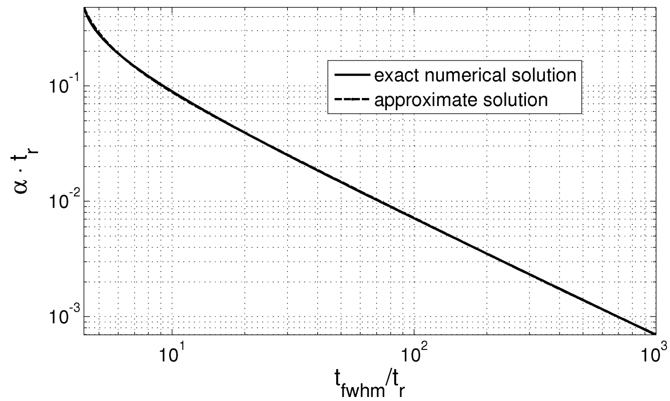

Fig. 3. Comparison of exact and approximated values of the parameter α normalized to t_r for different ratios of t_{fwhm} to t_r.

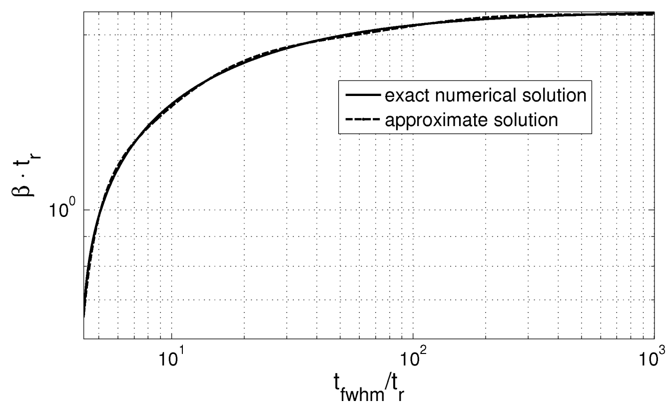

Fig. 4. Comparison of exact and approximated values of the parameter β normalized to t_r for different ratios of t_{fwhm} to t_r.

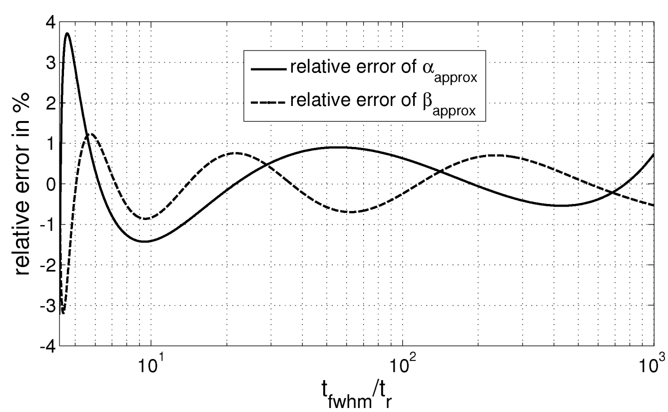

Fig. 5. Relative estimation errors of α and β for different ratios of t_{fwhm} to t_r.

respectively, in the interval $4.291 \leq \mu \leq 1000$. These estimation errors are plotted in Fig. 5. As compared to the approximation that was derived in [2], the relative error in this paper increased marginally, but the presented approach does not require a lookup table. The relative error is still lower than the error that results from the approximation formulas that were developed by the 2-D optimization in [3]. All discussed approximations are computationally much more efficient than solving the nonlinear system of equations numerically.

VI. Conclusion

This paper shows that the Nelder–Mead simplex algorithm in combination with the least-squares method is appropriate to determine the relationship between the commonly used physical parameters: rise time and pulsewidth, and the mathematical parameters α and β of double-exponential pulse shapes. With the help of these approximate correlations, it is possible to perform an extensive analysis of double-exponential pulse shapes with a very wide range of parameters in a considerably smaller computing time. The estimation errors of α and β are all less than 3.8%.

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CMOS OpAmp Resisting to Large Electromagnetic Interferences

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***Abstract*—A CMOS operational amplifier with high immunity to electromagnetic interferences is presented. It is based on an easy modification of the differential pair with active current load. The proposed input stage can be fabricated in standard CMOS technologies, and it neither requires extra mask levels, such as triple well, nor external components. Analysis and results are provided for very large interferences, which arise from the input pin.**

***Index Terms*—CMOS, immunity to electromagnetic interferences (EMIs), ICs, operational amplifier (OpAmp).**

I. Introduction

Due to the increasing adoption of electronic and microelectronic equipments, the immunity to electromagnetic interferences (EMIs) has become an important constraint for IC designers. The effects of EMI, indeed, may involve a wide class of circuits. Furthermore, the level of electromagnetic environmental pollution has been increasing during

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